JEE (MAIN)-2025 (Online)

Mathematics Memory Based Answer & Solutions

EVENING SHIFT

DATE : 29-01-2025

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MEMORY BASED QUESTIONS JEE–MAIN EXAMINATION – JANUARY, 2025

(Held On Wednesday 29th January, 2025) TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. If the letters of the word "KANPUR" are arranged in dictionary, then 440th word is

(1) PRKAUN (2) PRKUAN

(3) PRKNAU (4) PRKUNA

Ans. (1)

Sol. Letters starting with

$\underline{A}$ _ _ _ _ _ = 5! = 120

$\underline{K}$ _ _ _ _ _ = 5! = 120

$\underline{N}$ _ _ _ _ _ = 5! = 120

$\underline{P}\ \underline{A}$ _ _ _ _ = 4! = 24

$\underline{P}\ \underline{K}$ _ _ _ _ = 4! = 24

$\underline{P}\ \underline{N}$ _ _ _ _ = 4! = 24

$\underline{P}\ \underline{R}\ \underline{A}$ _ _ _ = 3! = 6

P R K A N U $\rightarrow$ 439th

P R K A U N $\rightarrow$ 440th

2. If 7^{103} is divided by 23, the remainder is

Sol. By FLT 23 is a prime number

$7^{22} \equiv 1 \pmod{23}$

$\Rightarrow 7^{103} = 7^{88+15} = 7^{88}.7^{15}$

$\Rightarrow 7^{103} \equiv 7^{15} \pmod{23}$

So, $(343)^5$

$7^{15} = (343)^5 = (345-2)^5$

$\Rightarrow 7^{15} = 23\lambda_1 + (-32)$

$= 23\lambda_2 - 32 + 96$

$= 23\lambda_2 + 14$

So, $7^{103} = 23\mu + 14$

3. Let $a_{ij} = (\sqrt{2})^{i+j}, A = [a_{ij}]_{3\times3}$. If sum of third row of A^2 is $\alpha + \beta\sqrt{2}$, then $\alpha+\beta$ is

Ans. (224)

Sol. $$A = \begin{bmatrix} 2 & 2\sqrt{2} & 4 \\ 2\sqrt{2} & 4 & 4\sqrt{2} \\ 4 & 4\sqrt{2} & 8 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} - & - & - \\ - & - & - \\ 56 & 56\sqrt{2} & 112 \end{bmatrix}$$

$\alpha + \beta\sqrt{2} = 168 + 56\sqrt{2}$

$\Rightarrow \alpha + \beta = 224$

4. Let $f(x) = \int_0^x t(t^2 - 3t + 20)dt, x \in [1,3]$ and range of $f(x)$ is $[\alpha, \beta]$, then $\alpha+\beta$ is equal to

Ans. $\left(\frac{185}{2}\right)$

Sol. $f'(x) = x(x^2 - 3x + 20) = x^3 - 3x^2 + 20x \uparrow x \in (1, 3)$

$\downarrow$

$D < 0$

$$f(1) = \int_0^1 (t^3 - 3t^2 + 20t)dt = \left(\frac{t^4}{4} - \frac{3t^3}{3} + \frac{20t^2}{2}\right)_0^1$$

$$\Rightarrow f(1) = \frac{1}{4} - 1 + 10 = \frac{37}{4} = \alpha$$

$$\Rightarrow f(3) = \int_0^3 (t^3 - 3t^2 + 20t)dt = \left(\frac{t^4}{4} - \frac{3t^3}{3} + \frac{20t^2}{2}\right)_0^3$$

$$= \frac{81}{4} - 27 + 10 \times 9 = \frac{333}{4} = \beta$$

$$\Rightarrow \alpha + \beta = \frac{37+333}{4} = \frac{370}{4} = \frac{185}{2}$$

5. The value of the limit

$$\lim_{x\to0}(\text{cosec}x)\left(\sqrt{2\cos^2x + 3\cos x} - \sqrt{\cos^2x + \sin x + 4}\right)$$

is

(1) 0 (2) 1

(3) $\frac{1}{2\sqrt{5}}$ (4) $-\frac{1}{2\sqrt{5}}$

Ans. (4)

Sol. $$\lim_{x\to0}\frac{2\cos^2x+3\cos x-\left(\cos^2x+\sin x+4\right)}{\sin x\left(\sqrt{2\cos^2x+3\cos x}+\sqrt{\cos^2x+\sin x+4}\right)}$$

$$\lim_{x\to0}\left(\frac{\cos^2x+3\cos x-4-\sin x}{\sin x}\right)\times\frac{1}{2\sqrt5}$$

$$=\frac{1}{2\sqrt5}\lim_{x\to0}\left(\frac{(\cos x+4)(\cos x-1)-\sin x}{\sin x}\right)$$

$$=-\frac{1}{2\sqrt5}\lim_{x\to0}\left(\frac{(\cos x+4)\times2\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$$

$$=-\frac{1}{2\sqrt5}\lim_{x\to0}\left(\frac{(\cos x+4)+\cos\frac{x}{2}}{\cos\frac{x}{2}}\right)=\frac{-1}{2\sqrt5}$$

6. $a_1, a_2, a_3, \ldots, a_{2024}$ are in A.P.

If $a_1+(a_5+a_{10}+a_{15}+\cdots+a_{2020})+a_{2024}=2233$, then find the value of $a_1+a_2+\cdots+a_{2024}$

Ans. (4)

Sol. $a_1+a_5+a_{10}+\ldots..+a_{2020}+a_{2024}$

$\Rightarrow 203(a_1+a_{2024})=2233$

$\Rightarrow a_1+a_{2024}=11$

$$S_{2024}=\frac{2024}{2}\left(a_1+a_{2024}\right)$$

$$\Rightarrow S_{2024}=(101)^2\times11=11132$$

7. Let the line L be $\frac{x-1}{1}=\frac{y-4}{3}=\frac{z-7}{5}$ and foot of perpendicular from $(1,-2,-1)$ to L is (α,β,γ), then the value of $\alpha+\beta+\gamma$ is

(1) $-\frac{69}{35}$ (2) $\frac{102}{35}$

(3) $\frac{69}{35}$ (4) $-\frac{102}{35}$

Ans. **(4)**

Sol. Any point on Lis $P(\lambda+1, 3\lambda+4, 5\lambda+7)$

Given $Q(1,-2,-1)$

Dr's of $PQ = <\lambda, 3\lambda+6, 5\lambda+8>$

$\overrightarrow{PQ}$ is Perpendicular to line L,

So, $\lambda\times1+(3\lambda+6)\times3+(5\lambda+8)\times5=0$

$$\Rightarrow \lambda=\frac{-58}{35}$$

Now sum of Coordinates of P is

$$\Rightarrow \lambda+1+3\lambda+4+5\lambda+7=9\lambda+12=\frac{-102}{35}$$

8. If the exhaustive values of a for which the equation $2x^2+(a-5)x+15=3a$ has no real roots is (α,β), then $|4(\alpha+\beta)|$ is equal to

(1) 56 (2) 52

(3) 54 (4) 18

Ans. **(1)**

Sol. $2x^2+(a-5)x+15=3a$

$\Rightarrow 2x^2+(a-5)x+(15-3a)=0$

$D<0$

$(a-5)^2-8(15-3a)<0$

$\Rightarrow a^2+14a-95<0$

$\Rightarrow (a+19)(a-5)<0$

$a\in(-19,5)$

$\therefore \alpha=-19$

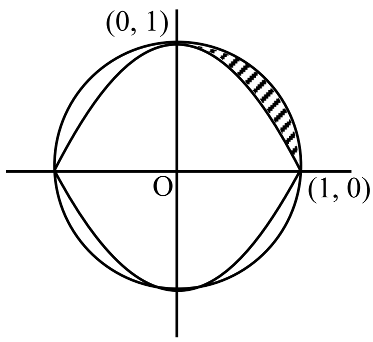
$\beta=5$

$\therefore \alpha+\beta=-14$

$\therefore |4(\alpha+\beta)|=56$

9. Let the area bounded by the curves $|y|=1-x^2$, $x^2+y^2=1$ is α. If $3\alpha=\beta\pi+\gamma$, then find $|\beta-\gamma|$

Sol.



$$\alpha=A=4\left[\frac{\pi}{4}-\int_0^1\left(1-x^2\right)dx\right]$$

$$\Rightarrow \alpha=4\left[\frac{\pi}{4}-\frac{2}{3}\right]$$

$$\Rightarrow \alpha = \pi - \frac{8}{3} \Rightarrow 3\alpha = 3\pi - 8$$

$$\beta = 3, \gamma = -8$$

$$|\beta - \gamma| = 11$$

10. If $x + y + z = 1$, $x + 2y + 4z = m$ & $x + 4y + 10z = m^2$ have infinite solutions and m takes 2 values α & β, then find

$$\sum_{r=1}^{10} (r)^{\alpha} + (r)^{\beta}$$

Ans. **(440)**

Sol. For infinite solutions $D = D_1 = D_2 = D_3 = 0$

$$\text{So, } D_1 = \begin{vmatrix} 1 & 1 & 1 \\ m & 2 & 4 \\ m^2 & 4 & 10 \end{vmatrix} = 0 \Rightarrow (m^2 - 3m + 2) = 0$$

$$\sum_{r=1}^{10}(r + r^2) = \sum_{r=1}^{10} r + \sum_{r=1}^{10} r^2$$

$$\Rightarrow \frac{10 \times 11}{2} + \frac{10 \times 11 \times 21}{6}$$

$$= 440$$

11. If $\log y = x \log \frac{2}{5}, x \in \mathbb{N} \cup \{0\}$. Then sum of all values of y is equals to

(1) $\frac{5}{3}$ (2) $\frac{2}{3}$

(3) $\frac{5}{4}$ (4) $\frac{8}{3}$

Ans. **(1)**

Sol. $\left(\log \frac{2}{5}\right) x = \log y$

$x = 0, y = 1$

$x = 1, y = \frac{2}{5}$

$x = 2, \ y = \left(\frac{2}{5}\right)^2$

$\vdots \qquad \vdots$

$\infty \qquad \infty$

$$\text{so, } \sum y = \frac{1}{1 - \frac{2}{5}} = \frac{5}{3}$$

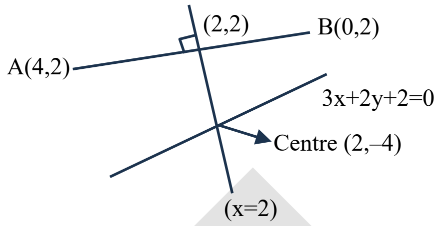
12. Two points (4 ,2) and (0, 2) lie on the circle whose centre lies on $3x + 2y + 2 = 0$, then length of the chord whose mid-point is (1, 2), is

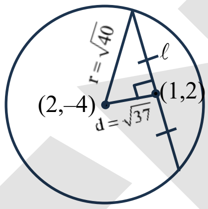
(1) $\sqrt{3}$ (2) $\sqrt{5}$

(3) $2\sqrt{3}$ (4) $2\sqrt{5}$

Ans. **(3)**

Sol.





$$r = \sqrt{40} = 2\sqrt{10}$$

$$\ell = 2\sqrt{r^2 - d^2} = 2\sqrt{40 - 37} = 2\sqrt{3}$$

13. The value of $\int_0^{\frac{\pi}{4}} \left(\sin\left|\left(4x - \frac{\pi}{2}\right)\right| + \sin[2x]\right) dx$ is

(where [.] denotes the greatest integer function)

(1) $\frac{1}{2} + \left(\frac{\pi-2}{4}\right)\sin 1$ (2) $\frac{1}{4} + \left(\frac{\pi-2}{2}\right)\sin 1$

(3) $\frac{1}{2} - \left(\frac{\pi-2}{4}\right)\sin 1$ (4) $\frac{1}{4} - \left(\frac{\pi-2}{2}\right)\sin 1$

Ans. **(1)**

Sol. $\int_0^{\frac{\pi}{4}} \left(\sin\left|4x - \frac{\pi}{2}\right| + \sin[2x]\right) dx$

$=$

$$\int_0^{\frac{\pi}{8}} -\sin\left(4x - \frac{\pi}{2}\right) dx + \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \sin\left(4x - \frac{\pi}{2}\right) dx + \int_0^{\frac{1}{2}} 0\, dx + \int_{\frac{1}{2}}^{\frac{\pi}{4}} \sin 1\, dx$$

$$= \left.\frac{\cos\left(4x - \frac{\pi}{2}\right)}{4}\right|_0^{\frac{\pi}{8}} - \left.\frac{\cos\left(4x - \frac{\pi}{2}\right)}{4}\right|_{\frac{\pi}{8}}^{\frac{\pi}{4}} + \left(\frac{\pi}{4} - \frac{1}{2}\right)\sin 1$$

$$= \left(\frac{1}{4} - 0\right) - \left(0 - \frac{1}{4}\right) + \left(\frac{\pi - 2}{4}\right)\sin 1$$

$$= \frac{1}{2} + \frac{(\pi - 2)\sin 1}{4}$$

14. If the domain of $log_{x-1}\left(\frac{2x^2-9x+4}{x^2-4x+5}\right)$ is (α,∞) and $log_5(18x-x^2-77)$ is (β,γ), then the value of $\alpha^2+\beta^2+\gamma^2$ is

Sol. $x - 1 > 0 \Rightarrow x > 1$

$$\& \ \frac{2x^2-9x+4}{x^2-4x+5} > 0$$

$x^2 - 4x + 5 > 0 \rightarrow D > 0$

and $2x^2 - 9x + 4 > 0$

$$\Rightarrow (x-4)\left(x-\frac{1}{2}\right) > 0$$

$\Rightarrow x \in (4, \infty) = (\alpha, \infty) \Rightarrow \alpha = 4$

and $18x - x^2 - 77 > 0$

$\Rightarrow x^2 - 18x + 77 < 0$

$\Rightarrow (x - 11)(x - 7) < 0$

$\Rightarrow x \in (7, 11) = (\beta, \gamma) \Rightarrow \beta = 7, \gamma = 11$

$\alpha^2 + \beta^2 + \gamma^2 = 16 + 49 + 121 = 186$

15. If $sinx + sin^2 x = 1$, then find the value of $cos^{12} x + tan^{12} x + 3[cos^8 x + tan^8 x] + cos^6 x + tan^6 x$

Sol. $\sin x = \cos^2 x \Rightarrow \sin^2 x + \sin x - 1 = 0$

$\cos^{12}x + \tan^{12}x + 3[\cos^8 x + \tan^8 x] + \cos^6 x + \tan^6 x$

Now,

$$\begin{array}{r|l} & 2\sin^6 x + 6\sin^4 x + 2\sin^3 x \\ \hline \sin^2 x + \sin x - 1 & 2\sin^6 x + 6\sin^4 x + 2\sin^3 x \\ & \underline{2\sin^6 x + 2\sin^5 x - 2\sin^4 x} \\ & -2\sin^5 x + 8\sin^4 x + 2\sin^3 x \\ & \underline{-2\sin^5 x - 2\sin^4 x + 2\sin^3 x} \\ & 10\sin^4 x \end{array}$$

$\because \sin^2 x + \sin x - 1 = 0$

$$\Rightarrow \sin x = \frac{-1+\sqrt{5}}{2}$$

$$\Rightarrow \sin^2 x = \frac{3-\sqrt{5}}{2}$$

$$\Rightarrow \sin^4 x = \left(\frac{7-3\sqrt{5}}{2}\right)$$

$\Rightarrow 10\sin^4 x = 35 - 15\sqrt{5}$

16. Bag 1 has 4 white and 5 black balls and bag 2 has n white and 3 black balls. A ball is chosen at random from bag 1 and put into bag 2, then a ball is drawn from bag 2. If probability of getting a white ball from bag 2 is $\frac{29}{45}$, then find the value of n

Sol. B_1 {4 white, 5 Black}

B_2 {n white, 3 Black}

$$P\left(\frac{W}{B_2}\right) = \frac{29}{45}$$

$\Rightarrow$ P(white ball from bag 1) × P(White ball from Bag 2) + P(Black ball from bag 1) × P(White ball from bag 2) = $\frac{29}{45}$

$$\Rightarrow \frac{4}{9}\times\frac{n+1}{n+4}+\frac{5}{9}\times\frac{n}{n+4}=\frac{29}{45}$$

$$\Rightarrow \frac{45}{9}(4n+4+5n) = 29(n+4)$$

$\Rightarrow 5(9n + 4) = 29n + 116$

$16n = 116 - 20$

$n = 6$

17. If $\lim_{t\to 0}\left(\int_0^1 (3x+5)^t dx\right)^{\frac{1}{t}} = \frac{\alpha}{5e}\cdot\left(\frac{8}{5}\right)^{\frac{2}{3}}$, then find the value of α

Ans. **(64)**

Sol.
$$\frac{\alpha}{5e} = \exp\left(\lim_{t\to 0}\frac{1}{t}\left(\int_0^1 (3x+5)^t dx - 1\right)\right)$$

$$= \exp\left(\lim_{t\to 0}\frac{1}{t}\left(\left.\frac{(3x+5)^{t+1}}{3(t+1)}\right|_0^1 - 1\right)\right)$$

$$= \exp\left(\lim_{t\to 0}\left(\frac{1}{t}\left(\frac{8^{t+1}-5^{t+1}}{3(t+1)} - 1\right)\right)\right)$$

$$= \exp\left(\lim_{t\to 0}\left(\frac{1}{t}\cdot\frac{8^{t+1}-5^{t+1}-3t-3}{3(t+1)}\right)\right)$$

$$= \exp\left(\lim_{t\to 0}\left(\frac{8^{t+1}\cdot\ln 8 - 5^{t+1}\ln 5 - 3}{3(t+1)}\right)\right)$$

$$= \exp\left(\frac{\ln 8^8 - \ln 5^5 - 3}{3}\right)$$

$$\Rightarrow \left(\frac{8}{3}\right)^{\frac{2}{3}}\frac{\alpha}{5e} = \exp\left(\frac{\ln\left(\frac{8^8}{5^5}\right)}{3} - 1\right)$$

$$\Rightarrow \left(\frac{8}{3}\right)^{\frac{2}{3}}\frac{\alpha}{5} = \left(\frac{8^8}{5^5}\right)^{\frac{1}{3}} = \left(\frac{8^6\cdot 8^2}{5^3\cdot 5^2}\right)^{1/3} = \frac{64}{5}\left(\frac{8}{5}\right)^{\frac{2}{3}}$$

$$\Rightarrow \alpha = 64$$

18. If $\frac{dy}{dx} + (\tan x)y = \frac{2+\sec x}{(1+2\sec x)^2}$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $f\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{10}$, then find the value of $f\left(\frac{\pi}{4}\right)$

Sol. $\text{I.F.} = e^{\int \tan x\, dx}$

$$= e^{\ln \sec x} = \sec x$$

$$\Rightarrow \sec x\frac{dy}{dx} + \sec x.\tan x.y = \frac{2+\sec x}{(1+2\sec x)^2}\sec x$$

$$\Rightarrow \sec x\, y = \int \frac{2\sec x + \sec^2 x}{(1+2\sec x)^2}dx$$

$$= \int\left(\frac{2\sec x + \sec^2 x}{\sec x \tan x}\right).\frac{\sec x \tan x}{(1+2\sec x)^2}dx$$

$$\int (2\cot x + \operatorname{cosec} x)\frac{-1}{2(1+2\sec x)}dx$$

$$\Rightarrow \frac{-(2\cot x + \operatorname{cosec} x)}{2(1+2\sec x)} - \int \frac{1}{\sin^2 x}.\frac{2+\cos x}{2(\cos x + 2)}.\cos x\, dx$$

$$\sec x.y = \frac{-(2\cot x + \operatorname{cosec} x)}{2(1+2\sec x)} + \frac{1}{2}\operatorname{cosec} x + c$$

$$x = \frac{\pi}{3}$$

$$2.\frac{\sqrt{3}}{10} = \frac{-\left(2.\frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}}\right)}{2(1+2.2)} + \frac{1}{2}.\frac{2}{\sqrt{3}} + c$$

$$\Rightarrow \frac{\sqrt{3}}{5} = \frac{-4}{\sqrt{3}}.\frac{1}{10} + \frac{1}{\sqrt{3}} + c$$

$$\Rightarrow c = \frac{\sqrt{3}}{5} + \frac{2}{5\sqrt{3}} - \frac{1}{\sqrt{3}}$$

$$= \frac{\sqrt{3}}{5} - \frac{\sqrt{3}}{5} = 0$$

19. The equation $\alpha x + \beta y = 109$ is a chord of ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ having midpoint $\left(\frac{5}{2}, \frac{1}{2}\right)$, then $\alpha + \beta$ is equal to

Ans. **(58)**

Sol. $T = S_1$

$$\Rightarrow \frac{5}{2}\cdot\frac{x}{9} + \frac{y}{4}\cdot\frac{1}{2} - 1 = \frac{\left(\frac{5}{2}\right)^2}{9} + \frac{\left(\frac{1}{2}\right)^2}{4} - 1$$

$$\Rightarrow \frac{5x}{18} + \frac{y}{8} - 1 = \frac{25}{36} + \frac{1}{16} - 1$$

$$\Rightarrow \frac{5x}{9} + \frac{y}{4} = \frac{25}{18} + \frac{1}{8}$$

$$\Rightarrow 20x + 9y = 50 + \frac{9}{2}$$

$$\Rightarrow 40x + 18y = 109$$

$$\Rightarrow \alpha + \beta = 58$$