JEE (MAIN)-2025 (Online)

Mathematics Memory Based
Answer & Solutions

MORNING SHIFT | **DATE : 29-01-2025**

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MEMORY BASED QUESTIONS JEE–MAIN EXAMINATION – JANUARY, 2025

(Held On Wednesday 29th January, 2025) **TIME : 9 : 00 AM to 12 : 00 PM**

MATHEMATICS	TEST PAPER WITH SOLUTION

SECTION-A

1. The minimum value of n for which the number of integer terms in the binomial expansion of $\left(7^{\frac{1}{3}} + 11^{\frac{1}{12}}\right)^n$ is 183, is

Sol. General term $\rightarrow {}^nC_r\left(7^{\frac{1}{3}}\right)^{n-r}\left(11^{\frac{1}{12}}\right)^r$

$$\Rightarrow {}^nC_r(7)^{\frac{n}{3}-\frac{r}{3}}(11)^{\frac{r}{12}}$$

For in integral terms, r must be multiple of 12.

i.e $r = 12\lambda$

given total values of $r = 183$

i.e. maximum value of $r = 12 \times 182 = 2184$

$\therefore$ Minimum value of $n = 2184$

2. $80\int_0^{\frac{\pi}{2}}\frac{\sin x + \cos x}{9+16\sin 2x}dx$

Sol. $80\int_0^{\frac{\pi}{2}}\frac{\sin x + \cos x}{9+16\sin 2x}dx$

$$= 80\int_0^{\frac{\pi}{2}}\frac{\sin x + \cos x}{9+16\{1-(1-\sin 2x)\}}dx$$

$$= 80\int_0^{\frac{\pi}{2}}\frac{\sin x + \cos x}{25-16(\sin x - \cos x)^2}dx$$

Put $\sin x - \cos x = t$

$(\cos x + \sin x)\,dx = 1dt$

$$= 80\int_{-1}^{1}\frac{1dt}{25-16t^2}$$

$$= \frac{80\times 2}{10}\,\frac{\log\left(\frac{5+4t}{5-4t}\right)\Big|_0^1}{4}$$

$$= 160\times\frac{1}{2\times 5}\,\frac{\ln\left|\frac{5+4t}{5-4t}\right|_0^1}{4}$$

$$= 4\left[\ln|9| - \ln|1|\right]$$

$$= 4\ln 9 = 8\ln 3$$

3. $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}, \vec{b} = 3\hat{i} - 5\hat{j} + \hat{k}$. If $\vec{a}\times\vec{c} = \vec{c}\times\vec{b}$ and $(\vec{a}+\vec{c})\cdot(\vec{b}+\vec{c}) = 168$, then |c|2 is equal to

Ans. **(308, 77)**

Sol. $\vec{a}\times\vec{c} = -\vec{b}\times\vec{c}$

$$\Rightarrow (\vec{a}+\vec{b})\times\vec{c} = \vec{0}$$

$$\therefore \vec{c} \parallel \vec{a}+\vec{b}$$

$$\vec{c} = \lambda\left(\vec{a}+\vec{b}\right) = \lambda\left(5\hat{i}-6\hat{j}+4\hat{k}\right)$$

$$(\vec{a}+\vec{c})\cdot(\vec{b}+\vec{c}) = 168$$

$$\Rightarrow \vec{a}\cdot\vec{b} + (\vec{a}+\vec{b})\cdot\vec{c} + |\vec{c}|^2 = 168$$

$$\Rightarrow 6+5+3+\lambda(25+36+16)+\lambda^2\times 77 = 168$$

$$14 + 77\lambda + 77\lambda^2 = 168$$

$$\Rightarrow \lambda^2 + \lambda - 2 = 0 \Rightarrow 1 = -2, 1$$

$$\therefore |\vec{c}|^2 = \lambda^2 \times 77$$

For $\lambda = -2$

$$|\vec{c}|^2 = 4\times 77 = 308$$

For $\lambda = 1$

$$|\vec{c}|^2 = 1\times 77 = 77$$

4. $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\dfrac{k^3+6k^2+11k+5}{(k+3)!}$ is equal to

(1) $\dfrac{5}{3}$ (2) $\dfrac{8}{3}$ (3) 3 (4) $\dfrac{7}{3}$

Ans. (1)

Sol. $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\dfrac{(k+1)(k+2)(k+3)-1}{\lfloor k+3}$

$\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\dfrac{1}{\lfloor k}-\dfrac{1}{\lfloor k+3}$

$=\left(\dfrac{1}{\lfloor 1}-\dfrac{1}{\lfloor 4}\right)+\left(\dfrac{1}{\lfloor 2}-\dfrac{1}{\lfloor 5}\right)+\left(\dfrac{1}{\lfloor 3}-\dfrac{1}{\lfloor 6}\right)+\left(\dfrac{1}{\lfloor 4}-\dfrac{1}{\lfloor 7}\right)+\ldots\ldots\ldots\ldots$

$=1+\dfrac{1}{\lfloor 2}+\dfrac{1}{\lfloor 3}$

$=1+\dfrac{1}{2}+\dfrac{1}{6}$

$=\dfrac{6+3+1}{6}=\dfrac{10}{6}=\dfrac{5}{3}$

5. $L_1:\dfrac{x-1}{1}=\dfrac{y-2}{-1}=\dfrac{z-1}{2},\ L_2:\dfrac{x+1}{-1}=\dfrac{y-2}{-2}=\dfrac{z}{1}$

Let the line L_3 passes through the point (α,β,γ) perpendicular to L_1& L_2 and L_3 intersect line L_1, then $|5\alpha-11\beta-8\gamma|$ is equal to

(1) 25 (2) 18 (3) 16 (4) 20

Ans. (1)

Sol. D. R. S of $L_3=\vec{m}\times\vec{n}$

$=-5\hat{i}-3\hat{j}+\hat{k}$

$=<5,3,-1>$

$\dfrac{x-\alpha}{5}=\dfrac{y-\beta}{3}=\dfrac{z-\gamma}{-1}=t$

$L_1=\dfrac{x-1}{1}=\dfrac{y-2}{-1}=\dfrac{z-1}{2}=k$

$5t+\alpha=k+1\Rightarrow\alpha=k+1-5t$

$3t+\beta=-k+2\Rightarrow\beta=-k+2-3t$

$-t+\gamma=2k+1\Rightarrow 2k+1+t$

$=\left|5(k+1-5t)-11(-k+2-3t)-8(2k+1+t)\right|$

$=|5-22-8|=25$

6. Sum of first three terms of an A.P. with integral common difference is 54 and sum of first twenty terms lies between 1600 to 1800, find a_{11}

(1) 108 (2) 90 (3) 111 (4) 115

Ans. (2)

Sol. $a+(a+d)+(a+2d)=54$

$3a+3d=54$

$a+d=18$

$a=18-d$ …(1)

$1600<\dfrac{20}{2}(2a+19d)<1800$

$160<2a+19d<180$

$160<36-2d+19d<180$

$124<17d<144$

$7.29<d<8.47$

$d=8$

from (1) $a=10$

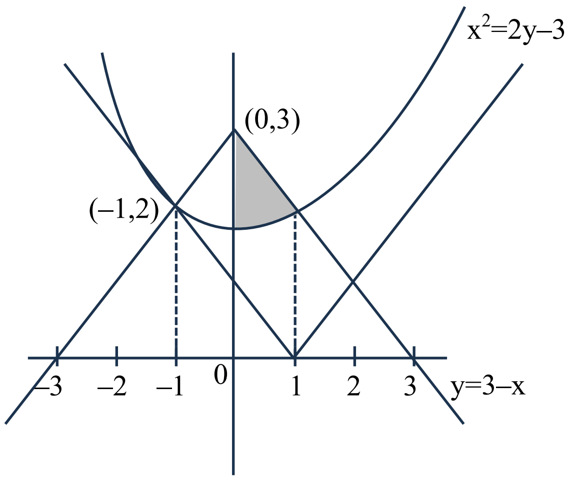
$a_{11}=a+10d$

$=10+80=90$

7. Area enclosed by $y\ge|x-1|,\ y+|x|\le 3, x^2\le 2y-3$ is A, then 6A (in sq. units)

Ans. (10)

Sol.



$A=2\left[\int\limits_0^1(3-x)dx-\int\limits_0^1\left(\dfrac{x^2+3}{2}\right)dx\right]$

$$A = 2\left[\left(3x - \frac{x^2}{2}\right)_0^1 - \frac{1}{2}\left[\frac{x^3}{3} + 3x\right]_0^1\right]$$

$$A = 2\left[\frac{5}{2} - \frac{1}{2}\left(\frac{10}{3}\right)\right]$$

$$A = 5 - \frac{10}{3} = \frac{5}{3}$$

6A = 10

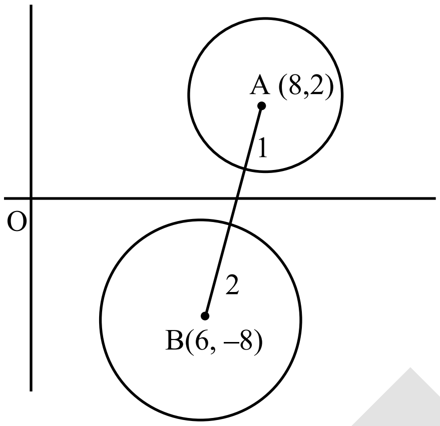
8. Let $|z_1 - 8 - 2i| \leq 1$ and $|z_2 - 6 + 8i| \leq 2$, and minimum value of $|z_1 - z_2|$ is equal to $\sqrt{a} - b$ where $a, b \in N$, then the value of $a + b$ is equal to

(1) 104 (2) 102

(3) 107 (4) 108

Ans. **(3)**

Sol.



$\because AB = \sqrt{104}$

$\therefore$ minimum value of $|z_1 - z_2| = \sqrt{104} - 3$

a + b = 104 + 3 = 107

9. If R be a relation defined on $\left(0, \frac{\pi}{2}\right)$ such that $xRy \Rightarrow \sec^2 x - \tan^2 y = 1$, then the relation R is

(1) Equivalence relation

(2) Reflexive and transitive only

(3) Symmetric and transitive only

(4) Neither reflexive nor transitive

Ans. **(1)**

Sol. (i) xRx

$\sec^2 x - \tan^2 x = 1$ Reflexive

(ii) $\sec^2 x - \tan^2 y = 1$

Then $\sec^2 y - \tan^2 x = 1 + \tan^2 y - (\sec^2 x - 1)$

$= 2 - 1 = 1$ Symmetric

(iii) $\sec^2 x - \tan^2 y = 1$

$\sec^2 y - \tan^2 z = 1$

add

$\sec^2 x - \tan^2 z = 1$ Transitive

Hence, equivalence relation.

10. Number of 7-digit numbers made with the digit 1, 2, 3 such that sum of the digits is 11 is equal to

Ans. **(161)**

Sol. (i) Number of numbers created using

$$1111133 = \frac{7!}{5!2!} = 21$$

(ii) Number of numbers created using

$$1111223 = \frac{7!}{4!2!} = 105$$

(iii) Number of numbers created using

$$1112222 = \frac{7!}{3!4!} = \frac{7 \cdot 5 \cdot 6}{6} = 35$$

Total = 161

11. If $\cos^{-1} x = \pi + \sin^{-1} x + \sin^{-1}(2x - 1)$, then find the sum of all values of x.

(1) 1 (2) $\frac{1}{2}$ (3) 0 (4) $\frac{3}{2}$

Ans. **(3)**

Sol. $\cos^{-1} x = \pi + \frac{\pi}{2} - \cos^{-1} x + \sin^{-1}(2x - 1)$

$$2\cos^{-1} x = \frac{3\pi}{2} + \sin^{-1}(2x - 1)$$

$$\cos\left(2\cos^{-1} x\right) = \cos\left(\frac{3\pi}{2} + \sin^{-1}(2x - 1)\right)$$

$$2x^2 - 1 = \sin\left(\sin^{-1}(2x - 1)\right)$$

$$2x^2 - 1 = 2x - 1$$

$x = 0, 1$

$x = 0$ is only one solution

But $x = 1$, is rejected

12. The minimum value of $p \in N$ such that

$$\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\ldots+\left[\frac{p}{x}\right]\right)-x^2\left(\left[\frac{1}{x^2}\right]+\left[\frac{2}{x^2}\right]+\ldots+\left[\frac{9}{x^2}\right]\right)\geq 1$$

is equal to

Ans. **(10)**

Sol. $$\lim_{x\to 0^+} x\left(\frac{1}{x}+\frac{2}{x}+\ldots\ldots+\frac{p}{x}\right)+x^2\left(\frac{1}{x^2}+\frac{2}{x^2}+\ldots\ldots+\frac{9}{x^2}\right)$$

$$-x\left(\left\{\frac{1}{x}\right\}+\left\{\frac{2}{x}\right\}+\ldots\ldots+\left\{\frac{p}{x}\right\}\right)+x^2\left(\left\{\frac{1}{x^2}\right\}+\ldots\ldots+\left\{\frac{9}{x^2}\right\}\right)\geq 1$$

$$\Rightarrow (1+2+3+\ldots\ldots+p)-\frac{9\times(9+1)}{2}\geq 1$$

$$\frac{p(p+1)}{2}\geq 46$$

Where $p \in N$ & min value $= 10$

13. If $L=\begin{vmatrix}\sin^2 x & 1+\cos^2 x & \sin 4x\\ 1+\sin^2 x & \cos^2 x & \sin 4x\\ \sin^2 x & \cos^2 x & 1+\sin 4x\end{vmatrix}$ and $L_{min} = m$ and $L_{max} = M$, then $|M^4 - m^4|$ is equal to

(1) 79 (2) 78 (3) 80 (4) 76

Ans. **(3)**

Sol. $$L=\begin{vmatrix}\sin^2 x & 1+\cos^2 x & \sin 4x\\ 1+\sin^2 x & \cos^2 x & \sin 4x\\ \sin^2 x & \cos^2 x & 1+\sin 4x\end{vmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$=\begin{vmatrix}\sin^2 x & 1+\cos^2 x & \sin 4x\\ 1 & -1 & 0\\ 0 & -1 & 1\end{vmatrix}$$

$= \sin^2x(-1) - (1 + \cos^2x)(1 - 0) + \sin4x(-1)$

$= -\sin^2x(-1) - 1 - \cos^2x - \sin4x$

$= -1 - 1 - \sin4x$

$= -2 - \sin4x$

$L_{min} = -2 - 1 = -3 = m$

$L_{max} = -2 + 1 = -1 = M$

$|M^4 - m^4| = |(-1)^4 - (-3)^4| = 80$

14. If α, β real numbers such that $\sec^2(\tan^{-1}\alpha) + \operatorname{cosec}^2(\cot^{-1}\beta) = 36$ and $\alpha + \beta = 8$, then $\alpha^3 + \beta^3$ is equal to

(1) 146 (2) 152 (3) 148 (4) 150

Ans. **(2)**

Sol. $\tan^{-1}\alpha = A$ $\qquad \alpha = \tan A$

$\cot^{-1}\beta = B$ $\qquad \beta = \cot B$

$\sec^2 A + \operatorname{cosec}^2 B = 36$

$1 + \tan^2 A + 1 + \cot^2 B = 36$

$\alpha^2 + \beta^2 = 34$

$\therefore \alpha^2 + \beta^2 + 2\alpha\beta = 64 \Rightarrow \alpha\beta = 15$

$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$

$= 8(34 - 15)$

$= 8(19) = 152$

15. How many 6 letter words can be formed using the word "MATHS" such that any letter can be used maximum two times?

(1) 6400 (2) 8100

(3) 10000 (4) 9824

Ans. **(2)**

Sol. MM AA TT HH SS

2 Alike, 4 Distinct : ${}^5C_1 \times 1 \dfrac{6!}{2!} = 1800$

2 Alike, 2 Alike, 2 Distinct :

$${}^5C_2 \times {}^3C_2 \times \frac{6!}{2!2!} = 5400$$

2 Alike, 2 Alike, 2 Alike : ${}^5C_3 \times \dfrac{6!}{2!2!2!} = 900$

Total = 8100

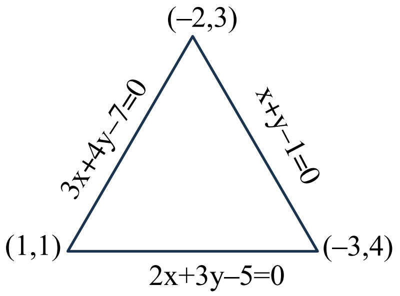
16. A triangle is formed by three lines $2x + 3y - 5 = 0$, $x + y - 1 = 0$, $3x + 4y - 7 = 0$. Let (h, k) be the image of the centroid of ΔABC in the line $2x + 4y - 7 = 0$, then $h^2 + k^2 + kh$ is equal to

(1) $\frac{903}{225}$ (2) $\frac{223}{225}$

(3) $\frac{100}{23}$ (4) $\frac{10006}{225}$

Ans. (1)

Sol.



$$\text{centroid} = \left(\frac{1+(-3)+(-2)}{3}, \frac{1+4+3}{3}\right)$$

$$= \left(-\frac{4}{3}, \frac{8}{3}\right)$$

Trianlge in $2x + 4y - 7 = 0$

$$\frac{k+\frac{4}{3}}{2} = \frac{k-\frac{8}{3}}{4} = -2\frac{\left(-\frac{5}{3}+\frac{32}{3}-7\right)}{4+16}$$

$$h = -\frac{23}{15}$$

$$k = \frac{34}{15}$$

$$h^2 + k^2 + hk = \left(-\frac{23}{15}\right)^2 + \left(\frac{34}{15}\right)^2 - \frac{23}{15} \times \frac{34}{15}$$

$$= \frac{903}{225}$$

17. Two parabolas having common focus at (4, 3) intersect at points A and B. Find the value of $(AB)^2$ given that directrices of these parabolas are along X-axis and Y-axis respectively.

Sol. $(x - 4)^2 + (y - 3)^2 = x^2$ …(1)

$(x - 4)^2 + (y - 3)^2 = y^2$ …(2)

$A(x_1, y_1)$ and $B(x_2, y_2)$

Both satisfy the above two equations

Also, from (1) & (2) we get that $x_1^2 = y_1^2$

$\Rightarrow x_1 = y_1$

So $(x_1 - 4)^2 + (x_1 - 3)^2 = x_1^2$ and

$(x_2 - 4)^2 + (x_2 - 3)^2 = x_2^2$

After solving

$x_1 + x_2 = 14$

$x_1 + x_2 = 25$

$$\text{So, } (AB)^2 = \left(\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}\right)^2$$

$$= 2(x_1 - x_2)^2$$

$$= 2\left[\sqrt{(x_1 + x_2)^2 - 4x_1x_2}\right]^2$$

$$= 2\left[\sqrt{(14)^2 - 4 \times 25}\right]^2$$

$= 2(196 - 100)$

$= 192$