JEE (MAIN)-2025 (Online)

Mathematics Memory Based
Answer & Solutions

MORNING SHIFT | **DATE : 23-01-2025**

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MEMORY BASED QUESTIONS JEE–MAIN EXAMINATION – JANUARY, 2025

(Held On Thursday 23rd January, 2025) TIME : 9 : 00 AM to 12 : 00 PM

MATHEMATICS	TEST PAPER WITH SOLUTION

SECTION-A

1. If for an arithmetic progression, if first term is 3 and sum of first four terms is equal to $\frac{1}{5}$ of the sum of next four terms, then the sum of first 20 terms is

(1) 1080 (2) 364

(3) –1080 (4) –364

Ans. (3)

Sol. $S_4 = \frac{1}{5}(T_5 + t_6 + t_7 + t_8)$

$$\frac{4}{2}(2a+3d) = \frac{1}{5}(4a+(4d+5d+6d+7d))$$

$10(2a + 3d) = 4a + 22d$

$16a = -8d$

$a = -\frac{d}{2}$

$$S_{20} = \frac{20}{2}(6+(19\times -6))$$

$=10(6 - 114) = -108 \times 10 = -1080$

2. How many words can be formed from the word DAUGHTER such that all vowels do not come together

(1) 34000 (2) 35000

(3) 36000 (4) 37000

Ans. (3)

Sol. D G H T R $\boxed{\text{AUE}}$

Total – (all values together)

$8! - (6! \times 3!) = 3600$

3. Two biased dice are tossed. Die 1 has 1 on two faces, 2 on two faces, 3 and 4 on other faces, while die 2 has 2 on 2 faces, 4 on 2 faces and 1 and 3 on

(1) $\frac{3}{7}$ (2) $\frac{2}{3}$

(3) $\frac{4}{9}$ (4) $\frac{8}{9}$

Ans. (3)

Sol. $D_2 = \{1,1, 2, 2, 3, 4\}$

$D_2 = \{2, 2, 4, 4, 1, 3\}$

Sum = 4 & 5

$\{(2, 2), (1, 3), (3, 1) + (2, 3), (3, 2), (1, 4), (4, 1)\}$

$$P \Rightarrow \left(\frac{2}{6}\times\frac{2}{6}\right)+\left(\frac{2}{6}\times\frac{1}{6}\right)+\left(\frac{1}{6}\times\frac{1}{6}\right)+$$

$$\left(\frac{2}{6}\times\frac{1}{6}\right)+\left(\frac{1}{6}\times\frac{2}{6}\right)+\left(\frac{2}{6}\times\frac{2}{6}\right)+\left(\frac{1}{6}\times\frac{1}{6}\right)$$

$$= \frac{4}{36}+\frac{2}{36}+\frac{1}{36}+\frac{2}{36}+\frac{2}{36}+\frac{4}{36}+\frac{1}{36}$$

$$= \frac{16}{36} = \frac{4}{9}$$

3. If f(x) is continuous at x = 0, where

$$f(x) = \begin{cases} \frac{2}{x}(\sin(k_1+1)x + \sin(k_2+1)x) & x < 0 \\ 4 & x = 0 \\ \frac{2}{x}\log\left[\frac{k_2x+1}{k_1x+1}\right] & x > 0 \end{cases}$$

Then $k_1^2 + k_2^2$ is

Ans. (2)

Sol. $2[(k_1 + 1) + (k_2 + 1)] = 4$

$k_1 + k_2 = 0$

$$\frac{2}{4}\log\left(\frac{k_2+1+1}{k_1+1+1}\right)$$

$$2\left(\frac{\log(k_2+1+1)}{h} - \frac{2\log(k_1+1+1)}{h}\right)$$

$2(k_2 - k_1) = 4$

$k_2 - k_1 = 2$

$k_2 = 1$

$k_1 = -1$

$k_1^2 + k_2^2 = 2$

4. $A = \{1, 2, 3, 4\}$. A relation R defined on set A is given by $R = \{(3, 3), (2, 1), (2, 3)\}$ then minimum no. of element added in R to make R an equivalence relation.

(1) 10 (2) 9 (3) 7 (4) 5

Ans. (3)

Sol. R = {(3, 3), (2, 1), (2, 3)}

For reflexive : (1, 1), (2, 2), (4, 4)

For symmetric : (1, 2), (3, 2)

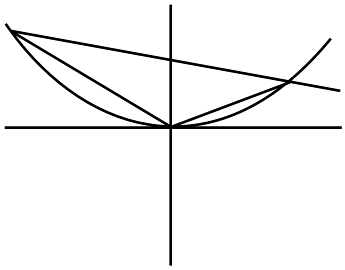
For transitive : (1, 3), (3, 1)

Minimum element added in R is 7

5. Find the angle subtended by the chord of parabola $2y = 3x^2$ intercepted by the line $x + y = 1$ at vertex.

Ans. $(\theta = \tan^{-1} 2\sqrt{7})$

Sol.



$3x^2 - 2y(x + y) = 0$

$3x^2 - 2xy - 2y^2 = 0$

$$\tan\theta = \frac{2\sqrt{1-3(-2)}}{3-2} = \frac{2\sqrt{h^2 - ab}}{a+b}$$

$= 2\sqrt{7}$

$\therefore\ \theta = \tan^{-1} 2\sqrt{7}$

6. If the curve satisfying the differential equation $\frac{dy}{dx} = \frac{6-2e^{2x}y}{1+e^{2x}}$ passes through (0, 0) and $(ln2,\ k)$, then k is

(1) $\frac{3}{5}ln3$ (2) $\frac{6}{5}ln2$

(3) $\frac{8}{9}ln3$ (4) $\frac{7}{2}ln2$

Ans. **(2)**

Sol. $$\frac{dy}{dx} + \frac{2y.e^{2x}}{1+e^{2x}} = \frac{6}{1+e^{2x}}$$

I.F. $e^{2\int \frac{e^{2x}dx}{1+e^{2x}}} = e^{\log(1+e^{2x})} = 1 + e^{2x}$

$y(1+e^{2x}) = \int 6.dx$

$y(1+e^{2x}) = 6x + c$

Passing through (0, 0) $\quad c = 0$

$y(1+e^{2x}) = 6x$

$$y = \frac{6x}{1+e^{2x}}$$

At $n = \ell n2$

$$k = \frac{6\ell n2}{1+4} = \frac{6}{5}\ell n2$$

7. Let $f(x) = \ell nx$ and $g(x) = \frac{x^4-2x^3+2x^2-x+1}{2x^2-2x+1}$, then domain of $f(g(x))$ for $x > 0$ is

(1) $(1, \infty)$ (2) $[1, \infty)$ (3) R (4) $(0, \infty) - \{1\}$

Ans. **(4)**

Sol. $x^4 - 2x^3 + 2x^2 - x + 1 > 0$

$(x^2+1)^2 - 2x(x^2+1) > 0$

$(x^2+1)(x^2-2x+1) > 0$

$(x^2+1)(x-1)^2 > 0$

$x > 0 \qquad x \in (0,1) \cup (1,\infty)$

8. $2(3+y)e^{2x}dx - (7+e^{2x})dy = 0$. Point (0, 5) satisfy the curve, also point $(\ell n2,\ k)$ satisfy the curve then $k =$

Ans. **(8)**

Sol. $$2(3+y)e^{2x}\frac{dx}{dy} = (7+e^{2x})$$

$e^{2x} = t$

$$2e^{2x}\frac{dx}{dy} = \frac{dt}{dy}$$

$$\Rightarrow (3+y)\frac{dt}{dy} = 7 + t$$

$$\Rightarrow \frac{dt}{7+t} = \frac{dy}{3+y}$$

$$\Rightarrow \ell n\frac{(7+t)}{3+y} = c$$

$$\Rightarrow \frac{7+e^{2x}}{3+y} = \alpha$$

$$\Rightarrow \frac{7+e^{2x}}{3+y} = \alpha$$

Point (0, 3)

$$\Rightarrow \frac{8}{8} = \alpha$$

$$\Rightarrow \alpha = 1$$

$$\Rightarrow 7 + e^{2x} = 3 + y$$

Put n = $\ell n 2$

$\Rightarrow 11 = 3 + y$

$\Rightarrow y = 2$

$\Rightarrow k = 8$

9. If $\left|\frac{\bar{z}+i}{2\bar{z}-i}\right| = \frac{1}{3}$, then the area bounded by centre of z, $(0, 0)$ and $(\alpha, 0)$ is 35, then 11 α is

Ans. (350)

Sol. Taking conjugate

$$\left|\frac{z-i}{2z+i}\right| = \frac{1}{3}$$

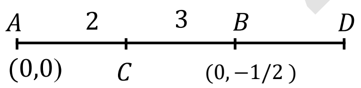
$$\Rightarrow \left|\frac{z-i}{z+\frac{i}{2}}\right| = \frac{2}{3}$$

$$C\left(0, \frac{2}{5}\right), D\left(0, \frac{-4}{-1}\right) = (0, 4)$$

$$\text{Centre} = \left(0, \frac{11}{5}\right)$$

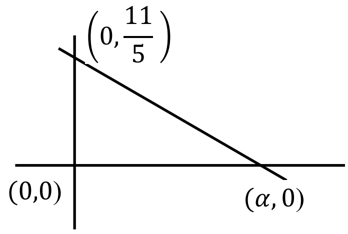
$$\frac{1}{2} \times \frac{11}{5} \times \alpha = 35$$

$$11\alpha = 350$$



$$\frac{CA}{CB} = \frac{2}{3} \text{ (internally)}$$

$$\frac{DA}{DB} = \frac{2}{3} \text{ (Externally)}$$



10. Value of $\cos^{-1}\left[\frac{12}{13}\cos x + \frac{5}{13}\sin x\right]$ is

$\left(x \in \left[\frac{\pi}{2}, \pi\right]\right)$

(1) $x + \tan^{-1}\frac{12}{13}$ (2) $x - \tan^{-1}\frac{12}{13}$

(3) $x - \tan^{-1}\frac{5}{12}$ (4) $x + \tan^{-1}\left(\frac{4}{5}\right)$

Ans. (3)

Sol. $\cos^{-1}[\cos\theta \cdot \cos x + \sin\theta \cdot \sin x]$

$= \cos^{-1}(\cos(x - \theta))$

$= x - \theta$

$x - \tan^{-1}\frac{5}{12}$

11. If for the system of linear equations having infinite solutions

$(\lambda - 4)x + (\lambda - 2)y + \lambda z = 0$

$2x + 3y + 5z = 0$

$x + 2y + 6z = 0$

Then $\lambda^2 + \lambda$ is

Ans. (90)

Sol. Hemogeneous equation

So, D = 0

$$\begin{vmatrix} \lambda - 4 & \lambda - 2 & \lambda \\ 2 & 3 & 5 \\ 1 & 2 & 6 \end{vmatrix} = 0$$

$\Rightarrow (\lambda - 4)(18 - 10) - (\lambda - 2)(12 - 5) + \lambda(4 - 3) = 0$

$\Rightarrow 2\lambda - 18 = 0$

$\lambda = 9$

So, $\lambda^2 + \lambda = 81 + 9 = 90$

12. If the equation $5x^3 - 15x + a = 0$ has three real roots then the range of $'a'$

Ans. (?)

Sol. f(x) = 5x³ – 15x + a

$f'(x) = 15x^2 - 15 = 0$

$f(1)f(-1) = (a - 10)(a + 10) \le 0$

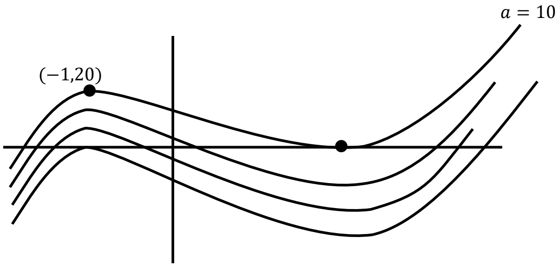
$-10 \le a \le 10$

$f(1) = 5 - 15 = 10 = 10$

$f(-1) = -5 + 15 + 10 = 20$

$= 15x - 5x^3$

$= 5x(3 - x^2)$



$f(x) = 5x^3 - 15x + 10$

$f(1) = 5 - 15 + 10 = 0$

$f(-1) = 5 + 15 + 10 = 20$

$f(x) = 5x^3 - 5x - 10$

$f(1) = 5 - 15 - 10 = -20$

$f(-1) = -5 + 15 + 10$

13. Function $f(x) = \log_e x$ and $g(x) = \frac{4}{2x^2-2x+1}$.

Domain of $y = f(g(x))$

Ans. (R)

Sol. $f(g(x)) = \ln g(x)$

$= \ln\left(\frac{4}{2x^2 - 2x + 1}\right)$

$2x^2 - 2x + 1 > 0$

$\forall\ x \in R$

Dom = R

14. Area of the larger region bounded by curves $y = |x - 1|$ and $x^2 + y^2 = 25$ is

Ans. $\left(\frac{75\pi}{4} + \frac{1}{2}\right)$

Sol. $\left\{\left(\frac{x}{2}\sqrt{5^2 - x^2} + \frac{5^2}{2}\sin^{-1}\frac{x}{5}\right)_{-3}^{4} - \frac{25}{3}\right\}$

$$25\pi - \left\{\left(2\times3\times\frac{25}{2}\times\sin^{-1}\frac{4}{5}\right) - \left(-\frac{3}{2}\times4 - \frac{25}{2}\sin^{-1}\frac{3}{5}\right)\right\} - \frac{25}{2}$$

$$25\pi - \left\{12 + \frac{25}{2}\times\frac{\pi}{2} - \frac{25}{2}\right\}$$

$$25\pi - \frac{25\pi}{4} + \frac{1}{2}$$

$$= \frac{75\pi}{4} + \frac{1}{2}$$

15. In the expansion of $\left(1 + 2^{\frac{1}{3}} + 3^{\frac{1}{2}}\right)^6$, find sum of all rational terms.

Ans. (612)

Sol. $(1 + 2^{1/3} + 3^{1/2})^6$

$\Rightarrow T_r = \frac{6!}{p!q!r!}(1)^p(2^{1/3})^q(3^{1/3})^r$

$\Rightarrow p + q + r = 6$

q = 0, 3, 6

p = 0, 2, 4, 6

so q = 3, 6, 0 r = 2, 4, 6, 0

q = 6, p = 0, r = 0

q = 0, r = 0, p = 6

r = 0, p = 4

r = 4, p = 2

r = 6, p = 0

q = 3 r = 0, p = 3

r = 2, p = 1

$$\text{sum} = \frac{6!}{6!}\times2^2 + \frac{6!}{6!}\times1 + \frac{6!}{2!4!}\times3\times1$$

$$+ \frac{6!}{4!2!}\times3^2\times1 + \frac{6!}{6!}\times3$$

$$+\frac{6!}{3!3!}\times 2\times 1+\frac{6!}{3!2!}\times 2\times 3$$

$= 1 \times 4 + 1 \times 1 + 15 \times 3 + 15 \times 9 + 1 \times 27 + 40 + 360$

$= 4 + 1 + 45 + 135 + 27 + 40 + 360$

$= 612$

16. Find the value of $\sin 70° (\cot 10° \cdot \cot 70° - 1)$

Ans. (1)

Sol. $\sin 70° (\cot 10° \cdot \cot 70° – 1)$

$$= \sin 70 \frac{(\cos 10 \cos 70 - \sin 10 \sin 70)}{\sin 10 \cdot \sin 70}$$

$$= \sin 70\left(\frac{\cos 80}{\sin 10 \sin 70}\right)$$

$$= \frac{\cos 80}{\sin 10} = \frac{\sin 10}{\sin 10} = 1$$

17. If $I(x) = \int \frac{dx}{(x-11)^{11/13}(x+15)^{15/13}}$ and

$I(37) - I(24) = \frac{1}{4}\left[\frac{1}{b^{\frac{1}{13}}} - \frac{1}{a^{\frac{1}{13}}}\right]$, then $3(a+b) =$

Ans. (39)

Sol. $$I(x) = \int \frac{dx}{(x-11)^{11/13}(x+15)^{15/13}}$$

$$= \int \frac{dx}{\left(\frac{x-11}{x+15}\right)^{11/13}(x+15)^2}$$

$$\frac{1}{26}\int t^{-11/13}dt$$

$$\frac{1}{26}\times\frac{t^{2/13}}{2/13}+c$$

$$I(x) = \frac{1}{4}\left(\frac{x-11}{x+15}\right)^{2/13}+c$$

$$I(37)-I(24)=\frac{1}{4}\left\{\left\{\frac{26}{52}\right\}^{2/13}-\left(\frac{13}{39}\right)^{2/13}\right\}$$

$$=\frac{1}{4}\left\{\left(\frac{1}{2}\right)^{2/13}-\left(\frac{1}{3}\right)^{2/13}\right\}$$

$$\frac{1}{4}\left\{\left(\frac{1}{4^{1/13}}-\frac{1}{9^{1/13}}\right)\right\}$$

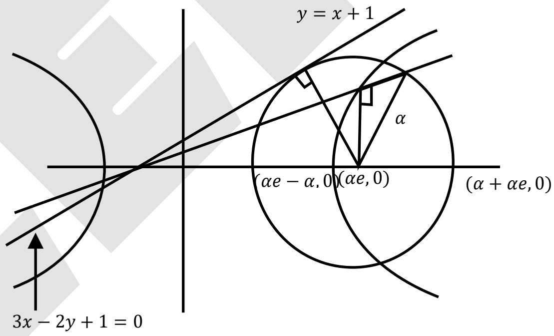
b = 4

a = 9

3(a + b) = 3(4 + 9) = 39

18. Centre of a circle lies on the positive x-axis such that centre coincides with focus of hyperbola $\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$ and diameter is equal to transverse axis, if equation of one of the tangent to circle is $x - y + 1 = 0$ and circle subtends a chord of length $\frac{4}{\sqrt{13}}$ on $3x - 2y + 1 = 0$ then find $3\alpha^2 + \beta^2$.

Ans. (25)



Sol.

$$\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$$

$$\left|\frac{\alpha e + 1}{\sqrt{2}}\right| = \alpha$$

$(\alpha e + 1)^2 = 2\alpha^2$

$\Rightarrow \alpha^2 e^2 + 2e\alpha + 1 = 2\alpha^2$ …(1)

$$\left|\frac{3\alpha e + 1}{\sqrt{9+4}}\right|^2 = \alpha^2 - \frac{4}{13}$$

$9\alpha^2 e^2 + 6e\alpha + 1 = 13\alpha^2 – 4$

$9\alpha^2 e^2 + 6e\alpha + 5 = 13\alpha^2$ …(2)

(1) = (2)

$$\frac{\alpha^2}{e^2}\ \frac{\alpha^2 e^2 + 2e\alpha + 1}{9\alpha^2 e^2 + 6e\alpha + 5} = \frac{2}{13}$$

$13\alpha^2 e^2 + 26e\alpha + 13 = 18 a^2 e^2 + 12e\alpha + 10$

$5\alpha^2 e^2 – 14e\alpha – 3 = 0$

$5\alpha^2 e^2 – 15e\alpha + e\alpha – 3 = 0$

$(5e\alpha + 1)(e\alpha - 3) = 0$

|

X

From (1) $(3 + 1)^2 = 2\alpha^2 \Rightarrow \alpha^2 = 8$

$$e = \frac{5}{2\sqrt{2}}$$

$\Rightarrow \alpha^2 e^2 = \alpha^2 + \beta^2$

$\beta^2 = 9 - 8 = 1$ $\qquad 3\alpha^2 + \beta^2 = 3 \times 8 \times 1 = 25$

$\beta = 1$

19. Value of $\displaystyle\int_{e^2}^{e^4} \left(\frac{1}{x}\right) \frac{e^{(1+(\ln x)^2)^{-1}}}{e^{(1+(\ln x)^2)^{-1}} + e^{(1+(6-\ln x)^2)^{-1}}} dx$ is

Ans. **(1)**

Sol. Put $\ln x = t \Rightarrow \frac{1}{x}dx = dt$

$$I = \int_2^4 \frac{e^{(1+t^2)^{-1}} dt}{e^{(1+t^2)^{-1}} + e^{(1+(6-t^2)^2)^{-1}}}$$

Apply kings

$$I = \int_2^4 \frac{e^{(1+(6-t)^2)^{-1}}}{e^{(1+(+-t)^2)^{-1}} + e^{(1+t^2)^{-1}}}$$

$$2I = \int_2^4 1 \cdot dx$$

$$2I = [X]_2^4$$

$\Rightarrow 2I = 2$

$I = 1$

20. Let A and B are non-singular commutative matrices. Then $A[(adjA^{-1}) \cdot (adj(B^{-1}))]^{-1}B$ is equal to

(1) $|A|\,|B|\,I_n$ $\qquad$ (2) $\frac{I_n}{|A||B|}$

(3) $\frac{I_n}{|A|}\frac{I_n}{|A|}$ $\qquad$ (4) $\frac{I_n}{|B|}$

Ans. **(1)**

Sol. $A\left\{\frac{A}{|A|} \cdot \frac{B}{|B|}\right\}^{-1} B$

$|A|\,|B|\,AB^{-1}A^{-1}B$

$\Rightarrow I\,|A|\,B|$

21. $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$

has equal roots and $a + c = 15$ and $b = \left(\frac{36}{5}\right)$

then the value of $a^2 + c^2 =$

Ans. **(117)**

Sol. b will be H.M. of a and c

$$\frac{ac}{15} = \frac{36}{5}$$

$ac = 54$

$9^2 + 6^2 = 81$

36

117

22. Foot of perpendicular of point Q(10,–3,–1) on line $L: \frac{x-3}{7} = \frac{y+1}{2} = \frac{z+2}{-1}$ is point P, then the area of ΔPQR when R is (3, 2, 1) is

Sol. Foot of $\perp$ of point

$Q(10, -3, -1)$ on line

$$L: \frac{x-3}{7} = \frac{y+1}{2} = \frac{z+2}{-1}$$

Is a point P, then the are

Of ΔPQR when R is (3, 2)

Let P(7k + 3, 2k – 1, -k, -2)

PQ. (7, 2, -1) = 0

(7 – 7k, -2, -2k, 1 + k)$\cdot$ (7, 2, -1) = 0

$\Rightarrow 49 - 49k - 4 - 4k - 1 - k = 0$

$\Rightarrow 44 = 54k$

$$\Rightarrow k = \frac{22}{27}$$

$\overrightarrow{PQ}(7 - 7k, -2 - 2k)$

$\overrightarrow{PR} = (-7k, 3 - 2k, 3 + k)$

$$= \frac{1}{2}\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right|$$

$$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7-7k & -2-2k & 1+k \\ -7k & 3-2k & 3+k \end{vmatrix}$$

$R_2 \to R_2 - R_3$

$$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -5 & -2 \\ -7k & 3-2k & 3+k \end{vmatrix}$$