JEE (MAIN)-2025 (Online)

Mathematics Memory Based Answer & Solutions

EVENING SHIFT

DATE : 23-01-2025

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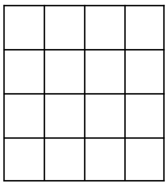
MEMORY BASED QUESTIONS JEE–MAIN EXAMINATION – JANUARY, 2025

(Held On Thursday 23rd January, 2025) TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS | TEST PAPER WITH SOLUTION

SECTION-A

1. If a square divided 4 × 4 squares. If two squares are chosen randomly then the probability that the squares doesn't share common side is



(1) 3/5 (2) 4/5
(3) 3/20 (4) 7/10

Ans. **(2)**

Sol. Probability $= \frac{{}^{16}C_2 - 24}{{}^{16}C_2} = \frac{120-24}{120} = \frac{96}{120} = \frac{4}{5}$

2. Find variance of numbers 8, 21, 34 ……., 320.

Sol. Method-1

Number of term 320 = 8 + (n – 1)13

312 = (n – 1)13

n – 1 = 24

n = 25

$\sigma^2 = \left(\frac{25^2-1}{12}\right)13^2$

$= \frac{24\times 26}{12}\times 13^2$

= 52 × 169 = 8788

Method-2

Mean $= \frac{8+21+34+....+320}{13} = \frac{\frac{13}{2}(8+320)}{13}$

$\frac{1}{13}\left(8^2+21^2+.....320^2\right)-164^2$

= 8788

3. There are 5 boys and 4 girls. The sum of number of ways to sit them such that all boys sit together and number of ways such that no boys sit together is equal to

Sol. 5B & 4G

Case-1

No boys together = 5! × 5! = 120 × 120

Case-2

All boys together = 4! × 5!

Total = 14400 + 24 × 120

= 120 × 144

= 17280

4. If the square of the shortest distance between the line $\frac{x-2}{1} = \frac{y-1}{2} = \frac{z+3}{3}$ and $\frac{x+1}{2} = \frac{y+3}{4} = \frac{z+5}{-5}$ is $\frac{m}{n}$, where m, n are coprime numbers then m + n is equal to ?

(1) 6 (2) 9 (3) 14 (4) 21

Ans. **(2)**

Sol. Ist line $\frac{x-2}{1} = \frac{y-1}{2} = \frac{z+3}{-3}$

Or $\vec{r} = 2i + j - 3k + \lambda(i+2j-3k)$

IInd line $\frac{x+1}{2} = \frac{y+3}{4} = \frac{z+5}{-5}$

$\vec{r} = -i-3j-5k+\mu(2u+4j-5k)$

$SD = \left|\frac{(\vec{a}-\vec{b}).(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|$

$= \frac{(3i+4j+2k).(2i-j)}{\sqrt{5}}$

$= \frac{6-4}{\sqrt{5}}$

$= \frac{2}{\sqrt{5}}$

∴ Square of shortest distance $= \frac{m}{n} = \frac{4}{5}$

∴ m = 4, n = 5

∴ m + n = 9

5. If $I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}}x\ dx}{\sin^{\frac{3}{2}}x + \cos^{\frac{3}{2}}x}$, then the value of definite integration $\int_0^{2I} \frac{x\sin x\cos x}{\sin^4 x + \cos^4 x}dx$ is

(1) $\frac{\pi}{16}$ (2) $\frac{\pi^2}{16}$ (3) $\frac{\pi}{8}$ (4) $\frac{\pi^2}{8}$

Sol. $I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}}x\ dx}{\sin^{\frac{3}{2}}x + \cos^{\frac{3}{2}}x}dx$

By property – 5

$$\int_0^{\frac{\pi}{2}} \frac{\cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$$

Let $J = \int_0^{2I} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$

$$J = \int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx \quad \ldots(i)$$

By property – 5

$$J = \int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2} - x\right) \sin x \cos x}{\sin^4 x + \cos^4 x} dx \quad \ldots(ii)$$

Do eq. (i) + (ii)

$$2I = \int_0^{\frac{\pi}{2}} dx$$

$$2I = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

6. $\frac{x^2}{2} + \frac{y^2}{4} = 1$, find length of chord with midpoint $\left(1, \frac{1}{2}\right)$ is

Sol. $\frac{x^2}{2} + \frac{y^2}{4} = 1 \quad \ldots(1)$

Equation of chord

$T = S_1$

$$\frac{x \times 1}{2} \rightarrow \frac{y \times \frac{1}{2}}{4} - 1 = \frac{1}{2} + \frac{1}{16} - 1$$

$$\frac{x}{2} + \frac{y}{3} = 1 \qquad \frac{x}{2} + \frac{y}{8} = \frac{9}{16}$$

$4x + y = 8 \qquad 8x + 2y = 9$

$$y = \frac{9 - 8x}{2} \quad \ldots(2)$$

From (1) & (2)

$$\Rightarrow \frac{x^2}{2} + \frac{1}{4}\left(\frac{9 - 8x}{2}\right)^2 = 1$$

$$\Rightarrow \frac{x^2}{2} + \frac{1}{16}\left(81 - 144x + 64x^2\right) = 1$$

$$\Rightarrow 8x^2 + 81 - 144 + 64x^2 = 16$$

$72x^2 - 144x + 65 = 0$

$x_1 + x_2 = 2$

$x_1 + x_2 = \frac{65}{72}$

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(x_1 - x_2)^2 + 16(x_1 - x_2)^2}$$

$$= |x_1 - x_2| \sqrt{17}$$

$$= \sqrt{2^2 - 4 \times \frac{65}{72}} \sqrt{17}$$

$$= \sqrt{4 - \frac{65}{18}} \sqrt{17}$$

$$= \frac{\sqrt{7}}{3\sqrt{2}} \times \sqrt{17} = \frac{\sqrt{119}}{3\sqrt{2}}$$

7. If $\int x^3 \sin x \, dx = g(x) + c$ where C is integration constant, $g(0) = 0$.

$\left(g\left(\frac{\pi}{2}\right) + g'\left(\frac{\pi}{2}\right)\right) = \alpha\pi^3 + \beta\pi^2 + \gamma$, then find the value of $\alpha + \beta - \gamma$

Sol. $\int x^3 \sin x dx$

$$= -x^3 \cos x + \int 3x^2 \cos x dx$$

$$= -x^3 \cos x + 3\left(x^2 \sin x - \int 2x \sin x dx\right)$$

$$= -x^3 \cos x + 3\left(x^2 \sin x - 2\left(-x \cos x + \int 1.\cos x dx\right)\right)$$

$$g(x) = -x^3 \cos x + 3\left(x^2 \sin x + 2x \cos x - 2 \sin x\right) + c$$

$$g(0) = 0 \Rightarrow c = 0$$

$$g\left(\frac{\pi}{2}\right) = 0 + 3\left(\frac{\pi}{2}\right)^2 + 0 - 2 \times 1$$

$$g'\left(\frac{\pi}{2}\right) = \left(\frac{\pi}{2}\right)^3$$

$$g\left(\frac{\pi}{2}\right) + g'\left(\frac{\pi}{2}\right) = \left(\frac{\pi}{2}\right)^3 + 3\left(\frac{\pi}{2}\right)^2 - 6$$

$$\alpha = \frac{1}{8}$$

$$\beta = \frac{3}{4}$$

$$\gamma = -6$$

$$\alpha + \beta - \gamma = \frac{1}{8} + \frac{3}{4} + 6 = \frac{1 + 6 + 48}{8} = \frac{55}{8}$$

8. α, β are the roots of quadratic equation $x^2 - px + q = 0$ and 10^{th} , 11^{th} terms of an A.P. whose common difference is $\frac{3}{2}$. Sum of 11 terms of the A.P. is 88, then find $q - 2p =$

Sol. $\alpha = a + 9d$

$\beta = a + 10d$

$$S_{11} = \frac{11}{2}(2a + 10d) = 88$$

$$a + 5d = 8$$

$$a + \frac{15}{2} = 8$$

$$a = 8 - \frac{15}{2}$$

$$a = \frac{1}{2}$$

$$\alpha = \frac{1}{2} + \frac{9 \times 3}{2} = \frac{28}{2} = 14$$

$$\beta = a + 10d = \frac{1}{2} + 15 = \frac{31}{2}$$

$\alpha + \beta = p$ $\qquad\qquad$ $\alpha\beta = q$

$14 + \frac{31}{2} = p$ $\qquad\qquad$ $14 \times \frac{31}{2} = q$

$\frac{59}{2} = p$ $\qquad\qquad$ $7 \times 31 = q$

$217 = q$

$q - 2p = 217 - 59 = 158$

9. If $|z| = 1$ and $\left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right| = 1$, then find the number of possible complex number.

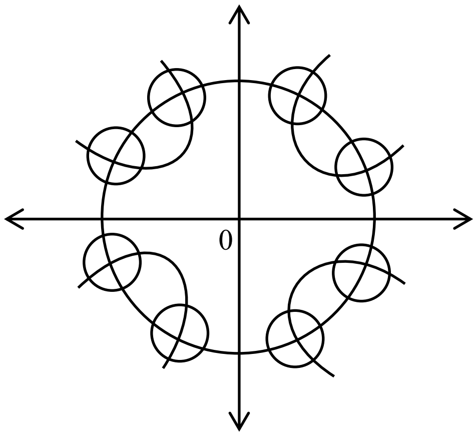
Sol. $x^2 + y^2 = 1$

$$\left|z^2 (\bar{z})^2\right| = 1$$

$$\left|x^2 + y^2 + x^2 + y^2\right| = 1$$

$$\left|x^2 - y^2\right| = \frac{1}{2}$$

$x^2 - y^2 = \frac{1}{2}$ $\qquad\qquad$ $x^2 - y^2 = -\frac{1}{2}$



10. Let S be the region consisting of points (x, y) such that $-1 \le x \le 1$

& $0 \le y \le a + e^{|x|} - e^{-|x|}$. If area bounded by the region is $2\left(\frac{e^2+8e+1}{e}\right)$ find "a".

Ans. (3)

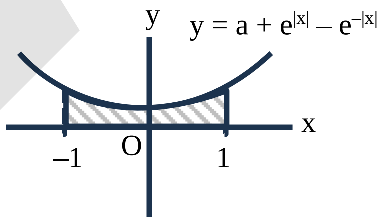
Sol. Let, $y = a + e^{|x|} - e^{-|x|}$

(1) $x \ge 0$, $\quad y = a + e^x - e^{-x}$

$\qquad y' = e^x + e^{-x}$

(2) $x \le 0$, $\quad y = a + e^{-x} - e^x$

$\qquad y' = -e^x - e^{-x}$



$$\text{Area} = 2\int_0^1 (a + e^x - e^{-x})dx$$

$$= 2\left[ax + e^x + e^{-x}\right]_0^1$$

$$2(e^2 + 8e + 1) = 2[(a + e + \frac{1}{e}) - (2)]$$

$$2(e^2 + 8e + 1) = 2ae + 2e^2 + 2 - 4e$$

$$e^2 + 8e + 1 = e^2 + (a - 2)e + 1$$

$\Rightarrow 8 = a - 2$

$\Rightarrow a = 10$

11. In the expansion of $(1+x)^p(1-x)^q$ coefficient of x and x^2 is 1and –2 then find p^2+q^2

Ans. **(13)**

Sol. coeff. of x $\Rightarrow$ p – q = 1 $\Rightarrow$ p = 1 + q

coeff. of $x^2 \Rightarrow$ ${}^qC_2 - pq + {}^pC_2 = -2$

$$\frac{q(q-1)}{2} - (1+q)q + \frac{(1+q)q}{2} = -2$$

$\Rightarrow q^2 - q - 2q - 2q^2 + q^2 + q = -4$

$\Rightarrow$ q = 2

p = 3

So, $p^2 + q^2 = 3^2 + 2^2 = 13$

12. Evaluate $\lim_{x\to\infty}\left(\frac{2x^2-3x+10}{3x^2+4x-2}\right)\frac{(3x-1)^{\frac{x}{2}}}{(\sqrt{3x+2})^x} =$

Ans. **(3)**

Sol. $$\lim_{x\to\infty}\frac{x^2\left(2-\frac{3}{x}+\frac{10}{x^2}\right)}{x^2\left(3+\frac{4}{x}-\frac{2}{x^2}\right)}\cdot\left(\frac{3x-1}{3x+2}\right)^{x/2}$$

$$\lim_{x\to\infty}\frac{2}{3}\times e^{\left(\frac{-3}{3x+2}\right)\left(\frac{x}{2}\right)}$$

$$= \frac{2}{3}e^{-\frac{1}{2}} = \frac{2}{3\sqrt{e}}$$

13. Let $f(x) = 6 + 16\cos\left(\frac{\pi}{3}-x\right)\cos\left(\frac{\pi}{3}+x\right)$ cosx sin3x cos6x. If range $f(x)$ is $[\alpha,\beta]$ then distance of point (α,β) from $3x+4y+12=0$ is

Ans. **(11)**

Sol. f(x) = 6 + $\frac{16}{4}$cos3x · sin 3x · cos6x

= 6 + 2 sin6x · cos6x

f(x) 6 + sin12x

f(x) = [5, 7]

3x + 4y + 12 = 0

$$d = \left|\frac{15+28+12}{5}\right| = \left|\frac{55}{5}\right| = 11$$

14. $A = (a_{ij})$ Given

$$A\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}0\\0\\1\end{bmatrix}, A\begin{bmatrix}4\\1\\3\end{bmatrix} = \begin{bmatrix}0\\1\\0\end{bmatrix}, A\begin{bmatrix}2\\1\\2\end{bmatrix} = \begin{bmatrix}1\\0\\0\end{bmatrix}.$$

Find (a_{23})

Ans. **(–1)**

Sol. $4a_{21} + 3a_{23} = 1$

$2a_{21} + 2a_{23} = 0$

$-4a_{23} + 3a_{23} = 1$

$a_{23} = -1$

15. The system of equations x + y + z = 6, x + 2y + 5z = 9, x + 5y + λz = μ has no solution, if

Ans. **λ = 17, μ = R – {18}**

Sol. x + y + z = 6

x + 2y + 5z = 9

x + 5y + λz = μ

$$D = \begin{vmatrix}1 & 1 & 1\\1 & 2 & 5\\1 & 5 & \lambda\end{vmatrix} = 1 - 17$$

$$D_1 = \begin{vmatrix}6 & 1 & 1\\9 & 2 & 5\\\mu & 5 & \lambda\end{vmatrix} = 3\lambda + 3\mu - 105$$

$$D_2 = \begin{vmatrix}1 & 6 & 1\\1 & 9 & 5\\1 & \mu & \lambda\end{vmatrix} = 3\lambda - 4\mu + 21$$

$$D_3 = \begin{vmatrix}1 & 1 & 6\\1 & 2 & 9\\1 & 5 & \mu\end{vmatrix} = \mu - 18$$

For no solution

D = 0

λ = 17

$D_1 = 3(\mu - 18)^3$

$D_3 = \mu - 18$

& $D_2 = 72 - 4\mu$

= 4(18 – μ)

λ = 17, μ = R – {18}

16. If $y=\left(x-y\frac{dx}{dy}\right)\sin\left(\frac{x}{y}\right)$ if $x(1)=\frac{\pi}{2}$ then find $\cos(x(2))$.

(1) ln2 (2) –ln2

(3) 2ln2 (4) $2\ln^2 2-1$

Ans. (4)

Sol. $y=\left(x-y\frac{dx}{dy}\right)\sin\left(\frac{x}{y}\right)$

$$1=\left(\frac{x}{y}-\frac{dx}{dy}\right)\sin\left(\frac{x}{y}\right)$$

$$\frac{x}{y}=v$$

$$\frac{dx}{dy}=v+y\frac{dv}{dy}$$

$$1=\left(v-\left(v+y\frac{dv}{dy}\right)\right)\sin v$$

$$i=v-v-y\frac{dv}{dy}.\sin x$$

$$1=-y\frac{dv}{dy}.\sin v$$

$$\frac{dy}{y}=-\sin vdy$$

$$\ln y=\cos v+c$$

$$\ln y=\cos\frac{x}{y}+c$$

$$0=0+c$$

$$c=0$$

$$\ln y=\cos\frac{x}{y}$$

$$\ln 2=\cos\left(\frac{x}{2}\right)$$

$$\cos x=2\cos^2\frac{x}{2}-1$$

$$=2\ln^2 2-1$$

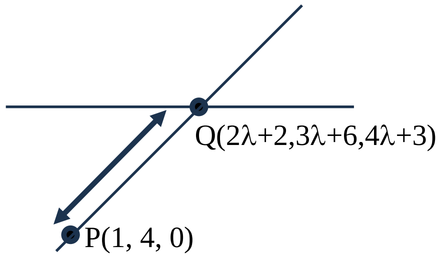
17. The distance of the line $\frac{x-2}{2}=\frac{y-6}{3}=\frac{z-3}{4}$ from the point (1, 4, 0) along the line $\frac{x}{1}=\frac{y-2}{2}=\frac{z+3}{3}$ is

(1) $\sqrt{14}$ (2) $\sqrt{17}$

(3) $\sqrt{13}$ (4) $\sqrt{15}$

Ans. (3)

Sol.



$$\frac{2\lambda+2-1}{1}=\frac{3\lambda+6-4}{2}=\frac{4\lambda+3-0}{3}$$

$$2\lambda+1=\frac{3\lambda+2}{2}=\frac{4\lambda+3}{3}$$

$4\lambda+2=3\lambda+2$ | $9\lambda+6=8\lambda+6$

$\lambda=0$ | $\lambda=0$

Distance = $\sqrt{(1)^2+(2)^2+(3)^2}$

$=\sqrt{14}$

18. Let P be the point $P(-1,-1,2)$ and $Q(5,5,10)$. A divide PQ in the ratio $\lambda:1$. If $(\overline{OQ}.\ \overline{OA})-\frac{1}{5}|OP\times OA|^2=10$, then find λ. (where O is origin).

(1) 5 (2) 7

(3) 9 (4) 12

Ans. (3)

Sol. P(–1. –1, 2) Q(5,5,10)

$$A\left(\frac{5\lambda-1}{\lambda+1},\frac{5\lambda-1}{\lambda+1},\frac{10\lambda+2}{\lambda+1}\right)$$

$$\overrightarrow{OQ}=(5,5,10)$$

$$\overrightarrow{OA}=\left(\frac{5\lambda-1}{\lambda+1},\frac{5\lambda-1}{\lambda+1},\frac{10\lambda-2}{\lambda+1}\right)$$

$$\overrightarrow{OP}=(-1,-1,2)$$

$$\overrightarrow{OQ}\cdot\overrightarrow{OA}=\frac{5}{\lambda+1}[5\lambda-1+5\lambda-1+20\lambda+4]$$

$$=\frac{5}{\lambda+1}[30\lambda+2]$$

$$\left|\overrightarrow{OP}\times\overrightarrow{OA}\right|^2=\frac{\lambda^2(20)^2}{(\lambda+1)^2}[2]$$

$$\frac{10}{\lambda+1}[15\lambda+1]-\frac{1}{5}\frac{\lambda^2(20)^2}{(\lambda+1)^2}[2]=10$$

$\Rightarrow (15\lambda + 1)(\lambda + 1) - \lambda^2(16) = (\lambda + 1)^2$

$\Rightarrow 2\lambda^2 - 14\lambda = 0$

$\lambda \neq 0$ so, $\lambda = 7$

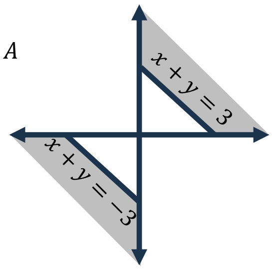
19. $A = \{(x, y) \mid |x + y| \geq 3\}$;

$B = \{(x, y) \mid |x| + |y| \leq 3\}$

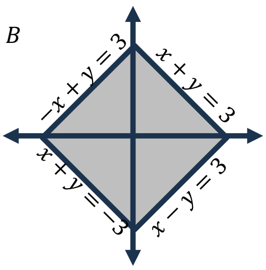
Let $C = A \cap B$. Find the sum of all values of $x + y \ \forall\, x, y \in C$.

Sol. **Ans. (0)**

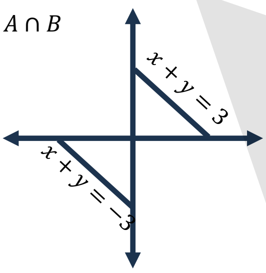
A



B



$A \cap B$



Ans. $3 - 3 = 0$