JEE (MAIN)-2025 (Online)

Mathematics Memory Based Answer & Solutions

MORNING SHIFT | **DATE : 22-01-2025**

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MEMORY BASED QUESTIONS JEE–MAIN EXAMINATION – JANUARY, 2025

(Held On Wednesday 22nd January, 2025) **TIME : 9 : 00 AM to 12 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. If $A = \{1, 2, 3\}$, find number of non-empty equivalent relation on set A.

(1) 4 (2) 5

(3) 6 (4) 7

Ans. (2)

Sol. $R_1 = \{(1, 1), (2, 2), (3, 3)\}$

$R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$

$R_3 = \{(1, 1), (2, 2), (3, 3), (1, 3), (3, 1)\}$

$R_4 = \{(1, 1), (2, 2), (3, 3), (2, 3), (3, 2)\}$

$R_5 = \{(1, 1), (2, 2), (3, 3), (1, 3), (2, 2), (1, 3), (3, 1), (2, 3), (3, 2)\}$

Total 5 relation are equivalence

2. A bag contains 6 white balls and 4 black balls, two balls are drawn at random, then the probability that both balls are white is -

(1) $\frac{1}{4}$ (2) $\frac{2}{3}$

(3) $\frac{1}{3}$ (4) $\frac{1}{2}$

Ans. (3)

Sol. $P(ww) = \frac{6}{10} \times \frac{5}{9} = \frac{1}{3}$

3. There are two lines L_1 and L_2 such that

$L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-1}{4}$ and $L_2 : \frac{x+2}{7} = \frac{y-2}{8} = \frac{z+1}{2}$.

Find the minimum distance between them

(1) $\frac{55}{\sqrt{1277}}$ (2) $\frac{66}{\sqrt{1277}}$

(3) $\frac{78}{\sqrt{1277}}$ (4) $\frac{88}{\sqrt{1277}}$

Ans. (4)

Sol. S.D. $= \left|\overrightarrow{AC}.\hat{n}\right|$

$A \equiv (1, 2, 1), C(-2, 2, 1)$

where $\hat{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 7 & 8 & 2 \end{vmatrix} = -26\hat{i} + 24\hat{j} - 5\hat{k}$

$= \left|\frac{(3\hat{i} + 2\hat{k})\cdot(26\hat{i} - 24\hat{j} + 5\hat{k})}{\sqrt{1277}}\right|$

$= \frac{78+10}{\sqrt{1277}} = \frac{88}{\sqrt{1277}}$

4. Coefficient of x^{2012} in the expansion of $(1 - x)^{2008}(1 + x + x^2)^{2007}$

(1) 0 (2) 1

(3) 2 (4) 4

Ans. (1)

Sol. $(1 - x)^{2008}(1 + x + x^2)^{2007}$

$= (1 - x^3)^{2007}\,(1 - x)$

$= \underbrace{(1 - x^3)^{2007}}_{T_1} - \underbrace{x(1 - x^3)^{2007}}_{T_2}$

To find coefficient of x^{2012}

From T_1

$T_{r+1} = {}^{2007}C_r\,(-x^3)^r$

$\Rightarrow 3r = 2012$

No such r is feasible

From T_2

$3r = 2011$

No such r is feasible

5. If $\frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$ and $x(1) = 1$. Then $x\left(\frac{1}{2}\right)$ is equal to .....

(1) 7 – e (2) 3 – e

(3) 5 – e (4) 11 – e

Ans. (2)

Sol. $\frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$

I.F. $= e^{\int \frac{1}{y^2}dy} = e^{-\frac{1}{y}}$

$\therefore\ x.e^{-\frac{1}{y}} = \int \frac{1}{y^3}.e^{-\frac{1}{y}}dy$

6. Let a coin is tossed thrice. Let the random variable X is tall follows a head and the mean of X is m and variance is s^2 respectively. The $64(\mu + \sigma^2)$ is

(1) 48 (2) 64

(3) 32 (4) 128

Ans. **(1)**

Sol.

Outcomes	X	P_i
HHH	0	$\frac{1}{8}$
HHT	1	$\frac{1}{8}$
HTH	1	$\frac{1}{8}$
THH	0	$\frac{1}{8}$
HTT	1	$\frac{1}{8}$
THT	1	$\frac{1}{8}$
TTH	0	$\frac{1}{8}$
TTT	0	$\frac{1}{8}$

$$\mu = \sum x_i p_i = 4 \times \frac{1}{8} = \frac{1}{2}$$

$$\sigma^2 = \sum x_i^2 p_i - \mu^2$$

$$= 4 \times \frac{1}{8} - \frac{1}{4} = \frac{1}{4}$$

$$64\left(\frac{1}{2} + \frac{1}{4}\right) = 64 \times \frac{3}{4} = 48$$

7. Let $g(x) = 3f\left(\frac{x}{3}\right) + f(3-x) \forall x \in (0,\ 3)$ and $f''(x) > 0 \forall x \in (0,\ 3)$, then $g(x)$ decreases in interval (0, a), then a is

(1) $\frac{7}{4}$ (2) $\frac{2}{3}$ (3) $\frac{9}{4}$ (4) $\frac{7}{3}$

Ans. **(3)**

Sol. Differentiating parent equation

$$g'(x) = 3f'\left(\frac{x}{3}\right) \times \frac{1}{3} + f'(3-x) \times -1$$

$$g'(x) = f'\left(\frac{x}{3}\right) - f'(3-x)$$

$$g'(x) < 0$$

$$g'(x) = f'\left(\frac{x}{3}\right) - f'(3-x) < 0$$

Since $f''(x) > 0$ so, f'(x) is increasing

$$\frac{x}{3} < 3 - x$$

$$\frac{4x}{3} < 3 \Rightarrow x < \frac{9}{4}$$

$$\alpha = \frac{9}{4}$$

8. If $f(x) = 16\left((\cos^{-1}x)^2 + (\sin^{-1}x)^2\right)$, then find sum of min. & max. value of $f(x)$

(1) $22\pi^2$ (2) $24\pi^2$

(3) $18\pi^2$ (4) $31\pi^2$

Ans. **(1)**

Sol. $f'(x) = 16\left(\frac{-2\cos^{-1}x}{\sqrt{1-x^2}} + \frac{2\sin^{-1}x}{\sqrt{1-x^2}}\right) = 0$

We are find the critical point

Case -1 $\sin^{-1} = \cos^{-1}x$

$x = \frac{1}{\sqrt{2}},\ y = 2\pi^2$

Case -2 $x = -1,$ $y = 20\pi^2$

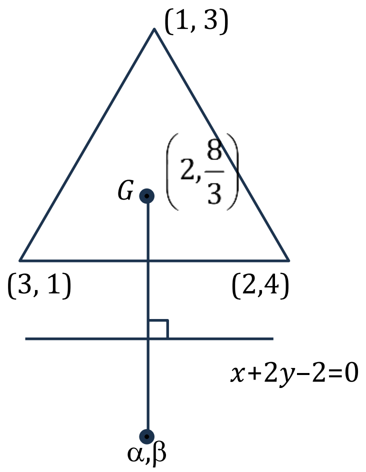
Case -3 $x = 1,$ $y = 4\pi^2$

Min + max = $22\pi^2$

SECTION-B

9. Let the triangle PQR be the image of the triangle with vertices (1, 3), (3, 1), (2, 4) in the line $x + 2y = 2$. If the centroid of ΔPQR is the point (α, β) then $15(\alpha - \beta)$ is equal to -

Ans. **(22)**



Sol.

$$\frac{\alpha-2}{1}=\frac{b-\frac{8}{3}}{2}=-2\frac{\left(2+\frac{16}{2}-2\right)}{1^2+2^2}$$

$$\frac{\alpha-2}{1}=\frac{b-\frac{8}{3}}{2}=-\frac{32}{15}$$

$$2-\frac{33}{15} = -2-\frac{2}{15}=\alpha$$

$$-\frac{64}{15}+\frac{8}{3}$$

$$-\frac{64+40}{15}=-\frac{24}{18}=\beta$$

$$15(\alpha-\beta)=15\left(-\frac{2}{15}+\frac{24}{15}\right)=22$$

10. If A be a 3 × 3 square matrix such that $\det(A) = -2$. If $\det(3\ \mathrm{adj}(-6\ \mathrm{adj}(3A))) = 2^n \times 3^m$, where $m \geq n$, that $4m + 2n$ is equal to -

Ans. **(104)**

Sol. $|A| = -2$, $\quad |3\mathrm{adj}(-6\mathrm{adj}(3A))|^2 = 27\ |-6\ \mathrm{adj}\ (3A)|^2$

$= 27 \cdot 6^6\ |\mathrm{adj}(3A)|^2$

$= 27 \cdot 6^6\ |3A|^4$

$27 \cdot 6^6 \cdot 3^{12}\ |A|^4$

$= 27 \cdot 6^6 \cdot 3^{12} \cdot 2^4$

$2^{10} \cdot 3^{21} \Rightarrow x = 10,\ m = 21$

$\Rightarrow 4\ m + 2n = 84 + 20 = 104$

11. A 5 letter word is to be made using any distinct 5 alphabets such that middle alphabet is M and letters should be in increasing order.

Ans. **$({}^{12}C_2 \times {}^{13}C_2)$**

Sol. **$({}^{12}C_2 \times {}^{13}C_2)$**

12. Two balls are selected at random one by one without replacement from the bag containing 4 white and 6 black balls. If the probability that the first selected ball is black given that the second selected is also black, is m/n where $\gcd(m,n) = 1$, then $m+n =$?

Ans. **(14)**

Sol. $P\left(\frac{\text{1st is as so black}}{\text{2nd is black}}\right)$

$$=\frac{\frac{6}{10}\times\frac{5}{9}}{\frac{4}{10}\times\frac{6}{9}+\frac{6}{10}\times\frac{5}{9}}$$

$$=\frac{30}{24+30}=\frac{30}{54}=\frac{5}{9}$$

$m + n = 5 + 9 = 14$

13. $y = x^2 + px - 3$ intersects coordinate axes at P, Q, R and a circle with center $(-1, -1)$ passing through P, Q, R is there find area of ΔPQR.

Ans. **(6)**

Sol. Family ≡ family ≡ $(x - \alpha)(x - \beta) + y^2 + \lambda y = 0$

It passes through $(0, -3)$

$\Rightarrow \alpha, \beta + 9 - 3\lambda = 0$

$-3 + 9 - 31 = 0 \Rightarrow \lambda = 2$

Circle equation ≡ $(x - \alpha)(x - \beta) + y^2 + 2y = 0$

$x^2 + px - 3 + y^2 + 2y = 0$

$\Rightarrow$ centre ≡ $\left(-\frac{p}{2}-1\right) \equiv (-1,-1) \Rightarrow p = 2$

Ar ΔPQR $\frac{1}{2}|\alpha-\beta|\times 3$

$= \frac{1}{2}\times\frac{\sqrt{D}}{|a|}\times 3$

$= \frac{1}{2}\times\sqrt{p^2+12}\times 3$

$\frac{1}{2}\times 4\times 3=6$

14. $\sum_{r=0}^{n} T_r = \frac{(2n-1)(2n+1)(2n+3)(2n+5)}{64}$

Find $\sum_{r=1}^{n}\frac{1}{T_r}$

Ans. $\left(\frac{2}{3}\left(\frac{4n^2+8n}{(2n+1)(2n+3)}\right)\right)$

Sol. $\frac{(2n-1)(2n+1)(2n+3)-(2n-3)(2n-1)(2n+1)(2n+3)}{64}$

$\frac{(2n-1)(2n+1)(2n+3)}{8}$

$\frac{8}{(2n-1)(2n+1)(2n+3)}$

$2\sum\frac{(2n+3)-(2n-1)}{(2n-1)(2n+1)(2n+3)}$

$= 2\sum_{1}^{n}\frac{1}{(2n-1)(2n+1)}-\frac{1}{(2n+1)(2n+3)}$

$2\left(\frac{1}{3}-\frac{1}{(2n+1)(2n+3)}\right)=\frac{2}{3}\left(\frac{4n^2+8n}{(2n+1)(2n+3)}\right)$

15. $e^{5(\ln x)^2+3} = x^8$. Product of all real values of x?

Ans. $(e^{8/5})$

Sol. $5(\log x)^3 + 3 ^{= 8\log e}x$

$\log_e x=\frac{3}{5}$ $\quad\log_e x=1$

$x=e^{\frac{3}{5}}$ $\quad x = e$

$e^{\frac{3}{5}}\times e^1 = e^{8/5}$

16. $\sum_{r=0}^{5}\frac{{}^{11}C_{2r-1}}{2r+2}=?$

Ans. $\left(\frac{1}{12}(2^{11}-1)\right)$

Sol. $\sum_{r=0}^{5}\frac{{}^{11}C_{2r+1}}{2r+2}$

$\Rightarrow\frac{1}{12}\sum_{r=0}^{5}{}^{12}C_{2r+2}$

$\Rightarrow\frac{1}{12}(12_{C_2}+12_{C_4}+12_{C_6}+12_{C_8}+$
$12_{C_{10}+12_{C_{12}}})$

$\Rightarrow\frac{1}{12}(2^{11}-1)$

17. Consider circle $(x-2\sqrt{3})^2+y^2=12$ & parabola $y^2=2\sqrt{3}x$. Find area bounded lies outside parabola and inside circle is……….

Ans. (2(3π-8))

Sol. Point of intersection of circle & Parabola

$= (2\sqrt{3},2\sqrt{3})$

Centre of circle $(2\sqrt{3},0)$

Rad = $2\sqrt{3}$

Area = $2\left[\frac{1}{4}\text{ area of circle }-\int_0^{2\sqrt{3}}\sqrt{2\sqrt{3}}.\sqrt{x}dx\right]$

Area = $2\left[\frac{1}{4}\cdot\pi\cdot 12-2^{\frac{1}{2}}\cdot 3^{\frac{1}{4}}\cdot\frac{2}{3}\cdot\left(x^{3/2}\right)_0^{2\sqrt{3}}\right]$

Area = 2(3π-8)

18. Let $f(x)$ be a real differentiable function such that $f(0)=1$ and $f(x+y)=f(x)f'(y)+f(y)f'(x)$ for all $x,\ y\in R$. Then $\sum_{n=1}^{100}\log_e f(n)=$

Ans. (2525)

Sol. $f(x+y)=f(x)f'(y)+f(y)f'(x)$ …(i)

Put y = 0 In (i)

f(x) = f(x) f'(0) + f'(x) …(ii)

put x = y = 0 in (i)

$f'(0)=\frac{1}{2}$ put in (ii)

$f'(x)=\frac{f(x)}{2}$

integrating

$$\ell n|f(x)| = \frac{1}{2}x + c$$

$$f(x) = e^{\frac{x}{2}}$$

$$\sum_{n=1}^{100} \log_e e^{\frac{n}{2}} = \sum_{n=1}^{100} \frac{n}{2} = 2525$$

19. If $A = \{1,2,\ldots,10\}$ and $B = \{\frac{M}{N} : M < N \;\&\; M, N \in A$ and $GCD(M,N) = 1\}$. Then $n(B) =$……

Ans. **(31)**

Sol. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\ldots \frac{1}{10} \rightarrow 9$

$\frac{2}{3}, \frac{2}{5}, \frac{2}{7}, \frac{2}{9} \rightarrow 4$

$\frac{2}{4}, \frac{3}{5}, \frac{3}{7}, \frac{3}{8}, \frac{3}{10} \rightarrow 5$

$\frac{4}{5}, \frac{4}{7}, \frac{4}{9} \rightarrow 3$

$\frac{5}{6}, \frac{5}{7}, \frac{5}{8}, \frac{5}{9}, \rightarrow 4$

$\frac{6}{7} \rightarrow 1$

$\frac{7}{8}, \frac{7}{9}, \frac{7}{10}, \rightarrow 3$

$\frac{8}{9} \rightarrow 1$

$\frac{9}{10} \rightarrow 1$

20. If L_1 & L_2 are two intersecting lines

$$L_1 : \frac{x-1}{2} = \frac{y}{0} = \frac{z-3}{\alpha}$$
$$L_2 : \frac{x}{1} = \frac{y}{2} = \frac{z-1}{0}$$

They intersects at B, if $A\,(-1, -1, 2)$ and P is its foot of perpendicular onto L_2 then find $26\alpha \times PB^2$

Ans. **(180)**

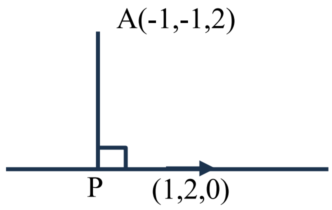
Sol. P.O.I of L_1 & L_2 is B

$(2k+1, 0, \alpha k + 3) = (\mu, 2\mu, 1)$

$\Rightarrow 2k + 1 = \mu | 0 = 2\mu | \alpha k + 3 = 1$

$\Rightarrow k = \frac{-1}{2}$ and $\mu = 0$ and $\alpha = 4$

So B is $(0,0,1)$



$\overrightarrow{AP}.(1,2,0) = 0$

$\Rightarrow \gamma + 1 + 4\gamma + 2 = 0$

$\Rightarrow \gamma = \frac{-3}{5}$

$26\alpha(PB)^2 = (26)(4)\left(\frac{9}{25} + \frac{36}{25}\right) = 180$

21. $f(x+y) = f(x)\,f(y)\;\forall\; x, y \in R$ and $f''(x) - 3af'(x) - f(x) = 0$ &
$f'(0) = 4a$ & $a > 0$ then find the area of $0 < y < f(ax)$ & $x \in (0,2)$

Ans. **(e^2–1)**

Sol. Let

$f(x) = p^x$

$f'(x) = p^x \ln p \Rightarrow f'(0) = 4a$

so $p = e^{4a}$

so $f(x) = e^{4ax}$

now,

$f''(x) - 3af'(x) - f(x) = 0$

Gives $a = \frac{1}{2}$

So $f(x) = e^{2x}$

Area of the region $0 < y < f(ax)$

$\Rightarrow 0 < y < e^x$

Area $= \int_0^2 e^x dx$

$= e^2 - 1$

22. If $8 = 3 + \frac{1}{4}(3 + P) + \frac{1}{4^2}(3 + p^2) + \ldots \infty$ then the value of p is

Ans. $(\frac{16}{5})$

Sol. $8=3\left[1+\frac{1}{4}+\frac{1}{4^2}+\ldots\right]+\frac{1}{4}\left[1+\left(\frac{p}{4}\right)+\left(\frac{p}{4}\right)^2+\ldots\infty\right]$

$\Rightarrow 8=3\times\left(\frac{1}{1-\frac{1}{4}}\right)+\frac{p}{4}\left[\frac{1}{1-\frac{p}{4}}\right]$

$\Rightarrow \frac{p}{4-p}=4$

$\Rightarrow 5p = 16$

$\Rightarrow p=\frac{16}{5}$

23. If $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$ if $I_1 = \int_0^{\frac{\pi}{4}} f(x)dx$, $I_2 = \int_0^{\frac{\pi}{4}} xf(x)dx$ then find $7I_1 + 12I_2$

Ans. (1)

Sol. $f(x)=\left(7\tan^6 x-3\tan^2 x\right)sec^2 x$

$I_1=\int_0^{\frac{\pi}{4}}\left(7\tan^6 x-3\tan^2 x\right)\sec^2 x dx$

Let tanx = t

$=\int_0^1\left(7t^6-3t^4\right)dt=\left(t^7-t^3\right)\Big|_0^1=0$

For $I_2=\int_0^{\frac{\pi}{4}} \underset{I}{x}\underbrace{\left(7\tan^6 x-3\tan^2 x\right)\sec^2 x}_{II} dx$

$\left(x\left(\tan^7 x-\tan^3 x\right)\right)\Big|_0^{\frac{\pi}{4}}-\int_0^{\frac{\pi}{4}}\left(\tan^7 x-\tan^3 x\right)dx$

$=0-\int_0^{\frac{\pi}{4}}\tan^3 x\left(\tan^2 x-1\right)\left(\tan^2+1\right)dx$

Let $\tan x=t$

$-\int_0^1\left(t^5-t^3\right)dt=-\left[\frac{t^6}{6}-\frac{t^4}{4}\right]_0^1=\frac{1}{12}$

$7I_1 + 12I_2 = 1$

24. Find the length of latus rectum of hyperbola such that the foci are (1, 14) and (1, –12) and passes through (1, 6)

Ans. (57.6)

Sol. $|SP - S'P| = 2a, SS' = 2ae$

$S(1,14)\ S'(1,-12)P(1,6)$

$\Rightarrow 2a = |8 - 18|$

$\Rightarrow a = 5, ae = 13$

Length of L.R.$=\frac{2b^2}{a}=\frac{288}{5}$

25. If $|z| = 1$ & z_1, z_2 z_3 lies on this circle. $\text{Arg}(z_1) = -\frac{\pi}{4}$ Arg $(z_2) = 0$ & Arg $(z_3) = \frac{\pi}{4}$ & If $|z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1|^2 = \alpha + \beta\sqrt{2}$ where α, β are integers the find α + β is

Ans. (3)

Sol.

$Z_1=\frac{1}{\sqrt{2}}(1+i), Z_2=1, Z_3=\frac{1}{\sqrt{2}}(1-i)$

$|\bar{Z}_1Z_2+\bar{Z}_2Z_3+\bar{Z}_3Z_1|$

$=\frac{1}{\sqrt{2}}(1+i)+\frac{1}{\sqrt{2}}(1-i)+\frac{1}{2}\times 2i|$

$=|\sqrt{2}+i(1-\sqrt{2})|$

$=\sqrt{2+1+2-2\sqrt{2}}$

$=\sqrt{5-2\sqrt{2}}$

$|\bar{Z}_1Z_2+\bar{Z}_2Z_3+\bar{Z}_3Z_1|^2=5-2\sqrt{2}=\alpha+\beta\sqrt{2}$

$\alpha=5, \beta=-2$

26. If $\vec{b} = \lambda\hat{\imath} + 4\hat{k}, \lambda > 0$ and the projection vector of $\vec{b}$ on $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} - \hat{k}$ is $\vec{c}$. If $|\vec{a} + \vec{c}| = 7$ then area of parallelogram formed by $\vec{b}$ and $\vec{c}$ is (in sq. units)

Ans. **(32)**

Sol. $\vec{a} + \vec{c} = (2\hat{\imath} + 2\hat{\jmath} - \hat{k})\left[1 + \frac{2(\lambda-2)}{9}\right]$

Given $|\vec{a} + \vec{c}| = 7$

$\Rightarrow 7 = 3\left[1 + \frac{2(\lambda-2)}{9}\right]$

$\Rightarrow \lambda = 8$

$\vec{c} = \frac{8\hat{\imath}+8\hat{\jmath}-4\hat{k}}{3}$

$\Rightarrow$ area of parallelogram $= |\vec{b} \times \vec{c}| = 32$