PHYSICS

1. (c)

Using tangent law

$$\frac{\mu_0}{4\pi}.\frac{2\pi nI}{r} = H\tan\theta$$

$nI \propto \tan\theta$

On increasing no. of turns, resistance and length increases so current decreases and nI remains same

2. (c)

Intensity=energy/sec/area=power/area.

From a point source, energy spreads over the surface of a sphere of radius$r$.

Intensity $=\frac{P}{A} = \frac{P}{4\pi r^2} \propto \frac{1}{r^2}$

But Intensity $= (\text{Amplitude})^2$

$\therefore (\text{Amplitude})^2 \propto \frac{1}{r^2}$ orAmplitude $\propto \frac{1}{r}$

At distance $2r$,amplitude becomes $A/2$

3. (d)

Intensity of electromagnetic wave is $I = \frac{P_{\text{av}}}{4\pi \times r^2} = \frac{E_0^2}{2\mu_0 c}$

or $E_0 = \sqrt{\frac{\mu_0 c P_{\text{av}}}{2\pi r^2}}$

$$= \sqrt{\frac{(4\pi \times 10^{-7}) \times (3 \times 10^8) \times 800}{2\pi \times (4)^2}}$$

$= 54.77 \text{ Vm}^{-1}$

4. (b)

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \Rightarrow T_2 = 300\left(\frac{27}{8}\right)^{\frac{5}{3}-1} = 300\left(\frac{27}{8}\right)^{\frac{2}{3}}$$

$$= 300\left\{\left(\frac{27}{8}\right)^{1/3}\right\}^2 = 300\left(\frac{3}{2}\right)^2 = 675K$$

$\Rightarrow \Delta T = 675 - 300 = 375K$

5. (b)

The average power consumed in an AC circuit is given by

$$P = \frac{V_0 I_0}{2}\cos\phi$$

Where $\phi$is phase angle and V_0 *and* I_0 the peak value of voltage and current.

Given, $V_0 = 200V, I_0 = 2\ A, \phi = \frac{\pi}{3}$.

$$P = \frac{200 \times 2}{2}\cos\frac{\pi}{3}$$

$$= \frac{200 \times 2}{2} \times \frac{1}{2} = 100 \text{ W}$$

6. (a)

Work done in moving a charge q into a uniform electric field E through a distance Y is

work done= Force $\times$ distance

$$= (qE) \times r = qEY$$

7. (b)

$\overline{A \cdot \overline{A}} = \overline{A} + \overline{\overline{A}} = \overline{A} + A = 1$

$A \cdot \overline{A} = 0$

$A + \overline{A} = 1$

$A + 1 = 1$

8. (b)

$Y = \frac{F}{A} \times \frac{L}{l}$ or $l = \frac{FL}{AY}$ or $l \propto 1/A$

9. (b)

Let 'a' be the retardation of boggy then distance covered by it be S. If u is the initial velocity of boggy after detaching from train ($i.e.$ uniform speed of train)

$v^2 = u^2 + 2as \Rightarrow 0 = u^2 - 2as \Rightarrow s_b = \frac{u^2}{2a}$

Time taken by boggy to stop

$v = u + at \Rightarrow 0 = u - at \Rightarrow t = \frac{u}{a}$

In this time t distance travelled by train $= s_t = ut = \frac{u^2}{a}$

Hence ratio $\frac{s_b}{s_1} = \frac{1}{2}$

10. (c)

According to Bernoulli's Theorem; $p = \frac{1}{2}\rho v^2$ =constant. Near the ends, the velocity of liquid is higher so that pressure is lower as a result the liquid rises at the sides to compensate for this drop of pressure

$ie, \rho\, g\, h = \frac{1}{2}\rho v^2 = \frac{1}{2}\rho r^2\omega^2$

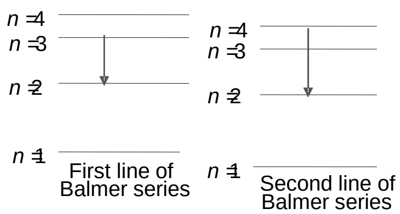
Hence, $h = \frac{r^2\omega^2}{2g} = \frac{r^2(2\pi v)^2}{2g} = \frac{2\pi^2 r^2 v^2}{g}$

$= \frac{2 \times \pi^2 \times (0.05)^2 \times 2^2}{9.8}$

$= 0.02\text{ m} = 2\text{ cm}$

11. (b)

12. (a)



For hydrogen or hydrogen type atoms

$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

In the transition from $ni \rightarrow nf$

$\therefore \quad \lambda \propto \frac{1}{Z^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$

$$\therefore \qquad \frac{\lambda_2}{\lambda_1} = \frac{Z_1^2\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)_1}{Z_2^2\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)_2}$$

$$\lambda_2 = \frac{\lambda_1 Z_1^2\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)_1}{Z_2^2\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)_2}$$

Substituting the values, we have

$$= \frac{(6561)(1)^2\left(\frac{1}{2^2}-\frac{1}{3^2}\right)}{(2)^2\left(\frac{1}{2^2}-\frac{1}{4^2}\right)} = 1215 \text{ Å}$$

13. (b)

As $\lambda_{\text{blue}} < \lambda_{\text{red}}$ and width of diffraction bands is directly proportional to λ, therefore diffraction bands become narrower and crowded

14. (b)

15. (a)

Apparent weight (w_a)= Actual weight (w)– upthrust(F), where upthrust = weight of water displaced = $V\, p\omega$ g

Now, $F_0 = V_0\rho_0$ g and $F_{50} = V_{50}\rho_{50}$g

$$\therefore \frac{F_{50}}{F_0} = \frac{V_{50}\rho_{50}\text{ g}}{V_0\rho_0\text{g}} = \frac{1+\gamma_m \times 50}{1+\gamma_w \times 50}$$

As $\gamma_m < \gamma_w$, therefore, $F_{50} < F_0$

Hence , $(w_a)_{50}(w_a)_0$or$w_2 > w_1$or $w_1 < w_2$

16. (c)

Time period $\qquad T = 2\pi\sqrt{\frac{m}{k}}$

$\because mg = kx$

$$\therefore T = 2\pi\sqrt{\frac{x}{\text{g}}}$$

$$(0.5)^2 = 4\pi^2 \times \sqrt{\frac{x}{10}}$$

$$\frac{(0.5)^2 \times 9.8}{4 \times 3.14 \times 3.14} = x$$

$x = 0.0621$ m

$x = 6.2$ cm

17. (b)

From the given circuit

$V_A - (6\times 2) - 12 - (9 \times 2) + 4 - (5 \times 2) = V_B$

Or $V_A - 12 - 12 - 18 + 4 - 10 = V_B$

Or $V_A - V_B$ =48 volt

18. (a)

When a piece of glass is heated, due to low thermal conductivity it does not conduct heat fast. Hence unequal expansion of it's layers crack the glass

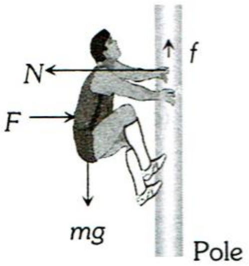
19. (c)

$\tau_{\max} = NiAB = 1 \times i \times (\pi r^2) \times B \quad \left[2\pi r = L, \Rightarrow r = \frac{L}{2\pi}\right]$

$\tau_{\max} = \pi\, i \left(\frac{L}{2\pi}\right)^2 B = \frac{L^2 iB}{4\pi}$

20. (c)

Here, $\mu = 0.8$. Let F be horizontal force that the body is applying on the pole. The various forces are acting on the boy as shown in the figure



Frictional force,

$f = \mu N = mg$

$N = \frac{mg}{\mu} = \frac{40 \times 10}{0.8} = 5000N \Rightarrow F = N = 500\ N$

21. (4)

Number of Nuclei decayed in time t,

$N_d = N_0(1 - e^{-\lambda t})$

$\therefore$ % decayed$=\left(\frac{N_d}{N_0}\right) \times 100$

$= (1 - e^{-\lambda t}d) \times 100$ ...(i)

Hence, $\lambda = \frac{0.693}{138} = 5 \times 10^{-4} s^{-1}$

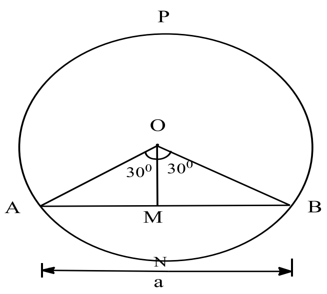
$\therefore$ % decayed $= (\lambda t) \times 100$

$= (5 \times 10^{-4})(80)(100)$

$= 4$

22. (6)

$ANBP$is cross-section of a cylinder of length L. The line charge passes through the centre O and perpendicular to paper.



$AM = \frac{a}{2}, MO = \frac{\sqrt{3}a}{2}$

$\therefore \angle AOM = \tan^{-1}\left(\frac{AM}{OM}\right)$

$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$

Electric flux passing from the whole cylinder

$\phi_1 = \frac{q_{\text{in}}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$

$\therefore$ Electric flux passing through $ABCD$ plane surface (shown only AB)= Electric flux passing through cylindrical surface ANB

$= \left(\frac{60°}{360°}\right)(\phi_1)$

$= \frac{\lambda L}{6\varepsilon_0}$

$\therefore n = 6$

23. (2)

$AB = 2\,R\cos\theta$

$AB = \frac{1}{2}g\cos\theta t^2 \Rightarrow 2\,R\cos\theta = \frac{1}{2}g\cos\theta t^2$

$2\sqrt{\frac{R}{g}} = t \Rightarrow 2\sqrt{\frac{10}{10}} = t = 2\text{s}$

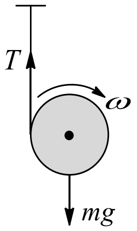
24. (8)

After long time, from conservation of momentum

$mv_0 = 2mv; v = \frac{v_0}{2} = 8\text{ m/s}$

25. (5)

Let T be the tension in the thread and f, the linear acceleration of the reel as it fails



For the downward translation

$(mg - T) = mf$ (i)

For the rotational motion of the reel, angular acceleration is $\alpha = \left(\frac{f}{a}\right)$and $T = \frac{mg}{3}$(ii)

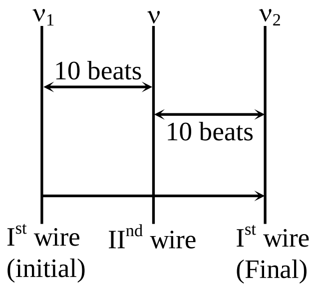
From Eqs (i) and (ii), $T = mg - mf = mg - ma\alpha$

$$= mg - 2T$$
$$\Rightarrow 3T = mg$$

$\therefore T = \frac{mg}{3} = 1.5 \times 10/3 = 5$ N

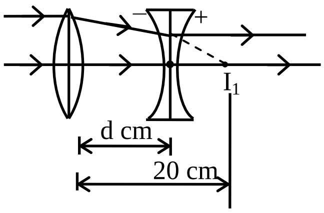
26. **Ans.** 0030
Sol. Self Teacher

27. **Ans.** 4
Sol. Frequency of first wire is less than that of second wire as upon increasing tension beats frequency remains unchanged.
$\therefore \nu = \nu_1 + 10 = 210$ Hz
$\therefore \nu - 206 = 4$



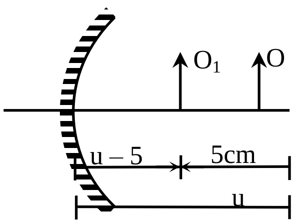
28. Sol. F = Ma ....(i)
$\frac{3}{2}$ Mg. 2R – F.R = $\frac{MR^2}{2} \cdot \frac{a}{R}$ ....(ii)
a = 2g
$\therefore$ F = 2Mg ≤ μN

29. **Ans.** 1
Sol.



V = ∞
U = + (20 – d)
f = –10
$$\frac{1}{-10} = \frac{1}{\infty} - \frac{1}{(20-d)}$$
$$-\frac{1}{10} = -\frac{1}{20-d}$$
20 – d = 10
d = 10 cm

30. **Ans.** 0002
Sol.



$m = -\frac{1}{4} \quad -\frac{v}{u} = -\frac{1}{4}$; $4v = u$ ...(1)

$\frac{1}{f}=\frac{1}{v_1}+\frac{1}{u_1}$; $m=-\frac{1}{2}$

$\frac{v_1}{u_1}=\frac{1}{2} \Rightarrow u_1=2v_1$

$u_1=3f$ …(2)

$\frac{1}{f}=\frac{1}{u}+\frac{1}{v} \Rightarrow \frac{1}{f}=\frac{5}{v}$

$u=5f$ …(3)

$u-u_1=5$

$5f-3f=5 \Rightarrow f=2.5$ cm

CHEMISTRY

1. (b)

The size of isoelectronic species increases with decrease in effective nuclear charge.

2. (b)

In addition homopolymers such as Teflon, empirical formula resembles with monomer.

3. (d)

4. (c)

d-block elements have higher melting point due to greater forces of attraction between two atoms.

5. (a)

The $H-O-H$ angle in water molecule is about 105° (due to two lone pairs of electrons)

6. (d)

Volume of the gold dispersed in one litre water

$$=\frac{\text{mass}}{\text{density}}=\frac{1.9\times10^{-4}\text{g}}{19\text{g cm}^{-3}}$$

$$=1\times10^{-5}\text{cm}^{-3}$$

Radius of gold sol particle =10 nm

$$=10\times10^{-9}\text{m}=10\times10^{-7}\text{cm}$$

Volume of the gold sol particle

$$=\frac{4}{3}\pi r^3$$

$$=\frac{4}{3}\times\frac{22}{7}\times(10^{-6})^3$$

$$=4.19\times10^{-18}\text{cm}^3$$

Number of gold sol particle in $1\times10^{-5}\text{cm}^3$

$$=\frac{1\times10^{-5}}{4.19\times10^{-18}}$$

$$=2.38\times10^{12}$$

Number of gold sol particle in one mm^3

$$=\frac{2.38\times10^{12}}{10^6}$$

$$=2.38\times10^6$$

7. (c)

Larger is the difference in electronegativities of two atom, more is polar character in bond.

8. (b)

$2 \times 1 + a + 4 \times (-2) = 0$
$\therefore a = +6$

9. (b)

Rest all replace —OH by —Cl.

10. (a)
11. (c)

Simple cations such as Ag^+, Cu^{2+}, Fe^{3+} etc. can accept pairs of electrons and hence are Lewis acids.

12. (b)

A derivation for mean free path of gas.

13. (b)

$2CaO + 2Cl_2 \longrightarrow CaCl_2 + Ca(ClO)_2$

14. (c)
15. (b)

No. of Na atoms present at each corner $= 8 \times \frac{1}{8} = 1$
No. of O atoms present at the centre of edges $= 12 \times \frac{1}{4} = 3$
No. of W atoms present at the centre of cube $= 1$
Formula of the compound $= NaWO_3$

16. (d)
17. (b)
18. (a)
19. (b)

$$N = \frac{4 \times 1000}{40 \times 100} = 1.0$$

20. (c)

It is a reason for given fact for given fact

21. (1)
22. (0)

All photochemical reactions have zero order of reaction

23. (6)
24. (4)
25. (2)
26. Sol. The maximum work done by the system

$= P_2 (V_2 - V_1) = P_2 \left(\frac{nRT}{P_2} - \frac{nRT}{P_1}\right) = mgh$

or $mgh = nRT \left(1 - \frac{P_2}{P_1}\right)$

or mass lifted, $m = \frac{nRT}{gh}\left(1 - \frac{P_2}{P_1}\right)$

$\therefore$ Mass lifted during expansion = m

$= \frac{nRT}{gh}\left(1 - \frac{P_2}{P_1}\right)$ Work done on the system during the compression

$= P_1 (V_2 - V_1) = P_1 \left(\frac{nRT}{P_2} - \frac{nRT}{P_1}\right) = nRT\left(\frac{P_1}{P_2} - 1\right)$

Let net mass lowered during the compression = m′

$\therefore m'gh = nRT\left(\frac{P_1}{P_2} - 1\right)$

or $m' = \frac{nRT}{gh}\left(\frac{P_1}{P_2} - 1\right)$

$\therefore$ net mass lowered through a height h = m"

$\therefore m'' = m' - m = \frac{nRT}{gh} \times \frac{(P_1 - P_2)^2}{P_2 P_1}$

$(P_1 - P_2)^2$ is always positive hence m" is positive.

27. Ans. 4

28. Ans. 750 Å

29. Ans. 1

Sol. $\Delta U = Q + W$

$\Rightarrow \Delta U = Q_V$ (W = 0 at constant volume)

$= m \times s \times \Delta t$ (m = mass of gas

s = specific heat)

$= 0.5 \times 40 \times (20.814 - 8.314) \times 4$ Joule

= 1 KJ

30. Ans.0833

MATHS

1. (d)

(a) It is clear that A is not a zero matrix.

(b) $(-1)I = -1\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}-1 & 0 & 0\\0 & -1 & 0\\0 & 0 & -1\end{bmatrix} \neq A$

$ie, \quad (-1)I \neq A$

(c) $|A| = 0\begin{vmatrix}-1 & 0\\0 & 0\end{vmatrix} - 0\begin{vmatrix}0 & 0\\-1 & 0\end{vmatrix} - 1\begin{vmatrix}0 & -1\\-1 & 0\end{vmatrix}$

$= 0 - 0 - 1(-1) = 1$

Since, $|A| \neq 0$ so A^{-1} exists.

(d) $A^2 = \begin{bmatrix}0 & 0 & -1\\0 & -1 & 0\\-1 & 0 & 0\end{bmatrix}\begin{bmatrix}0 & 0 & -1\\0 & -1 & 0\\-1 & 0 & 0\end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow A^2 = I$

2. (d)

We have,

$\tan 9° - \tan 27° - \tan 63° + \tan 81°$

$= (\tan 9° + \tan 81°) - (\tan 27° + \tan 63°)$

$= (\tan 9° + \cot 9°) - (\tan 27° + \cot 27°)$

$= \frac{1}{\sin 9° \cos 9°} - \frac{1}{\sin 27° \cos 27°}$

$= \frac{2}{\sin 18°} - \frac{2}{\sin 54°}$

$= 2\frac{\sin 54° - \sin 18°}{\sin 54° \sin 18°} = 2\frac{\cos 36° - \sin 18°}{\sin 18° \cos 36°} = 4$

3. (a)

$\because$ 26 cards can be chosen out of 52 cards in ${}^{52}C_{26}$ ways. There are two ways in which each card can be dealt because a card can be either from the first pack or from the second.

$\therefore$Total number of ways= ${}^{52}C_{26}.2^{26}$

4. (b)

Required ratio is given by

$-\frac{3 \times 1 + 3 - 9}{3 \times 2 + 7 - 9}$

$= \frac{3}{4}$ ie, 3: 4 internally

5. (a)

We have,

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{f(x) + f(h) - f(x)}{h} [\because f(x+y) = f(x) + f(y)]$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{f(h)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\sin h \ g(h)}{h} = \lim_{h \to 0} \frac{\sin h}{h} \lim_{h \to 0} g(h) = g(0) = k$

6. (b)

We have,

$\int \frac{a^{\sqrt{x}}}{\sqrt{x}} dx = 2\int \frac{a^{\sqrt{x}}}{2\sqrt{x}} dx = 2\int a^{\sqrt{x}} d(\sqrt{x}) = \frac{2a^{\sqrt{x}}}{\log a} + C$

7. (c)

Given, $y = \sin^{-1} x$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$ ...(i)

$\Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{0 - \frac{1}{2}.\frac{(-2x)}{\sqrt{1-x^2}}}{\left(\sqrt{1-x^2}\right)^2}$

$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} = x\,\frac{dy}{dx}$....[from Eq.(i)]

8. (c)

9. (a)

$x^y = e^{2(x-y)}$

$\therefore y\,\log x = 2(x-y)$

$\Rightarrow\ y\,(\log x + 2) = 2x$

$y = \dfrac{2x}{\log x + 2}$

$\dfrac{dy}{dx} = \dfrac{(\log x + 2)(2) - 2x.\frac{1}{x}}{(\log x + 2)^2}$

$= \dfrac{2\log x + 4 - 2}{(\log x + 2)^2} = \dfrac{2(\log x + 1)}{(\log x + 2)^2}$

10. (b)

Given limiting points are $(1, 2), (-2, 1)$

The mid point is $\left(-\frac{1}{2}, \frac{3}{2}\right)$

Now, slope$= \frac{1-2}{-2-1} = \frac{1}{3}$

$\therefore$ required equation, $y - \frac{3}{2} = -3\left(x + \frac{1}{2}\right)$

$\Rightarrow\ 3x + y = 0$

11. (c)

Since,

AM $\geq$ GM

$\Rightarrow \dfrac{a+b}{2} \geq \sqrt{ab}$

$\Rightarrow \frac{a+b}{2} \geq \sqrt{4}$ $(\because ab = 4$, given$)$

$\Rightarrow a + b \geq 4$

12. (d)

Given, $f(x) = ax + \frac{b}{x}$

On differentiating w.r.t. x, we get

$f'(x) = a - \dfrac{b}{x^2}$

For maxima or minima, put $f'(x) = 0$

$\Rightarrow\ x = \sqrt{\dfrac{b}{a}}$

Again, differentiating w.r.t. x, we get

$f''(x) = \dfrac{2b}{x^3}$

At $x = \sqrt{\frac{b}{a}}$, $f''(x) = +ve$

$\Rightarrow \; f(x)$is minimum at$x = \sqrt{\frac{b}{a}}$

$\therefore \; f(x)$ has the least value at $x = \sqrt{\frac{b}{a}}$

13. (d)

Given, $\tan^{-1}(x-1) + \tan^{-1} x = \tan^{-1} 3x - \tan^{-1}(x+1)$

$\Rightarrow \; \tan^{-1}\left[\frac{(x-1)+x}{1-(x-1)x}\right] = \tan^{-1}\left[\frac{3x-(x+1)}{1+3x(x+1)}\right]$

$\Rightarrow \; (1+3x^2+3x)(2x-1) = (1-x^2+x)(2x-1)$

$\Rightarrow \; (2x-1)(4x^2+2x) = 0$

$\Rightarrow \; x = 0, \pm\frac{1}{2}$

14. (b)

Let the vertices of a triangle be $A(6,0), B(0,6)$ and $C(6,6)$

Now, $AB = \sqrt{6^2+6^2} = 6\sqrt{2}$

$BC + \sqrt{6^2+0} = 6$

And $CA = \sqrt{0+6^2} = 6$

Also, $AB^2 = BC^2 + CA^2$

Therefore, ΔABC is right angled at$C$. So, mid point of AB is the circumcentre of ΔABC

$\therefore$ Coordinate of circumcentre are (3, 3)

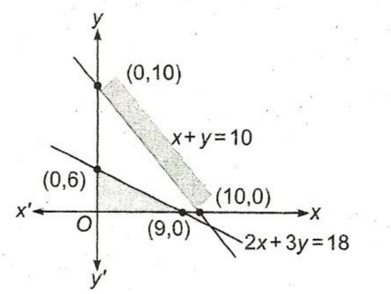
Coordinates of centroid are,

$G\left(\frac{6+0+6}{3}, \frac{0+6+6}{3}\right), ie, (4,4)$

$\therefore$ Required distance $= \sqrt{(4-3)^2+(4-3)^2} = \sqrt{2}$

15. (d)

From the figure it is clear that there is no common area. So, we cannot find maximum value of z



16. (c)

We have,

$p \leftrightarrow q \cong (p \to q) \wedge (q \to p)$

$\cong (\sim p \vee q) \wedge (\sim q \vee p)$

$\therefore \sim (p \leftrightarrow q) \cong \sim (\sim p \vee q) \vee \sim (\sim q \vee p)$

$\cong (p \wedge \sim q) \vee (q \wedge \sim p)$

17. (d)

Equation of the line passing through $(5,1,a)$and $(3,b,1)$ is

$$\frac{x-3}{5-3}=\frac{y-b}{1-b}=\frac{z-1}{a-1}\ldots..(i)$$

Also, point $\left(0,\frac{17}{2},-\frac{13}{2}\right)$ satisfies Eq. (i), we get

$$-\frac{3}{2}=\frac{\frac{17}{2}-b}{1-b}=\frac{-\frac{13}{2}-1}{a-1}$$

From Ist and IIIrd terms $a-1=\frac{\left(-\frac{15}{2}\right)}{\left(-\frac{3}{2}\right)}\Rightarrow a=6$

From Ist and IIIed terms $-3(1-b)=2\left(\frac{17}{2}-b\right)\Rightarrow b=4$

18. (a)

$^{50}C_4+{}^{55}C_3+{}^{54}C_3+{}^{53}C_3+{}^{52}C_3+{}^{51}C_3+{}^{50}C_3$
$={}^{51}C_4+{}^{51}C_3+{}^{52}C_3+{}^{53}C_3+{}^{54}C_3+{}^{55}C_3$
$[\because {}^{n}C_r+{}^{n}C_{r-1}={}^{n+1}C_r]$
$={}^{52}C_4+{}^{52}C_3+{}^{53}C_3+{}^{54}C_3+{}^{55}C_3$
$={}^{53}C_4+{}^{53}C_3+{}^{54}C_3+{}^{55}C_3$
$={}^{54}C_4+{}^{54}C_3+{}^{55}C_3={}^{55}C_4+{}^{55}C_3={}^{56}C_4$

19. (c)

Given $P(A\cup B)=0.6, P(A\cap B)=0.3$
$\therefore\ P(A')+P(B')$
$=1-P(A)+1-P(B)=2-\{P(A)+P(B)\}$
$=2-\{P(A\cup B)+P(A\cap B)\}$
$=2-\{0.6+0.3\}=2-0.9=1.1$**(c)**
Given $P(A\cup B)=0.6, P(A\cap B)=0.3$
$\therefore\ P(A')+P(B')$
$=1-P(A)+1-P(B)=2-\{P(A)+P(B)\}$
$=2-\{P(A\cup B)+P(A\cap B)\}$
$=2-\{0.6+0.3\}=2-0.9=1.1$

20. (b)

Since, α and β the roots of the equation $x^2-x-1=0$
$\therefore\ \alpha+\beta=1$ and $\alpha\beta=-1$
Hence, AM of $A_{n-1}$and$A_n=\frac{A_{n-1}+A_n}{2}$

$$=\frac{\alpha^{n-1}+\beta^{n-1}+\alpha^n+\beta^n}{2}$$

$$=\frac{\alpha^{n-1}(1+\alpha)+\beta^{n-1}(1+\beta)}{2}$$

$$=\frac{\alpha^{n-1}.\alpha^2+\beta^{n-1}\beta^2}{2}$$

$$=\frac{1}{2}(\alpha^{n+1}+\beta^{n+1})$$

$$=\frac{1}{2}A^{n+1}$$

21. (9)

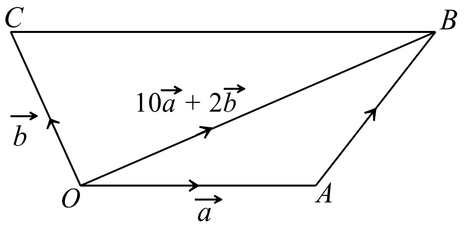
Given $f(x+2)=f(x)+f(2)$
Put $x=-1$, we have $f(1)=f(-1)+f(2)$
$\Rightarrow f(1)=-f(1)+f(2)$ (as $f(x)$ is an odd function)
$\Rightarrow f(2)=2f(1)=6$
Now put $x=1$,
We have $f(3)=f(1)+f(2)=3+6=9$

22. (0)

a, b, c are in A.P>$\Rightarrow b=\frac{a+c}{2}$ (1)
b, c, d are in G.P. $\Rightarrow c^2=bd$ (2)
And c, d, e are in H.P. $\Rightarrow d=\frac{2ce}{c+e}$ (3)
Now $c^2=bd \Rightarrow c^2=\left(\frac{a+c}{2}\right)\left(\frac{2ce}{c+e}\right)$ [using (1) and (3]
$\therefore c^2+ce=ae+ce$
$\Rightarrow c^2=ae$
Now given $a=2$ and $e=18$
$\therefore c^2=ae \Rightarrow c^2=2\times 18=36 \Rightarrow c=6$ or -6

23. (6)

Here $\vec{O}A=\vec{a}, \vec{O}B=10\,\vec{a}+2\vec{b}, \vec{O}C=\vec{b}$
$q=$ Area of parallelogram with OA and OC as adjacent sides
$\therefore q=|\vec{a}\times\vec{b}|$ (i)



$p=$ Area of quadrilateral $OABC$
$=$ Area of ΔOAB + Area of ΔOBC
$=\frac{1}{2}\left|\vec{a}\times\left(10\vec{a}+2\vec{b}\right)\right|+\frac{1}{2}\left|\left(10\vec{a}+2\vec{b}\right)\times\vec{b}\right|$
$=\left|\vec{a}\times\vec{b}\right|+5\left|\vec{a}\times\vec{b}\right|$
$\therefore p=6\left|\vec{a}\times\vec{b}\right|$
Or $p=6\,q$ [From Eq. (i)]
$\therefore k=6$

24. (3)

$x+y+z=1(1)$
$x+2y+4z=p$ (2)
$x+4y+10z=p^2$ (3)
$\Delta=\begin{vmatrix}1 & 1 & 1\\ 1 & 2 & 4\\ 1 & 4 & 10\end{vmatrix}$
$R_1\rightarrow R_1-R_2, R_2\rightarrow R_2-R_3$

$$= \begin{vmatrix} 0 & -1 & -3 \\ 0 & -2 & -6 \\ 1 & 4 & 10 \end{vmatrix} = 0$$

Since $\Delta = 0$, solution is not unique solution.

The system will have infinite solutions if $\Delta_1 = 0, \Delta_2 = 0, \Delta_3 = 0$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ p & 2 & 4 \\ p^2 & 4 & 10 \end{vmatrix} = 0$$

$C_3 \to C_3 - C_2$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 0 \\ p & 2 & 2 \\ p^2 & 4 & 6 \end{vmatrix} = 0$$

$\Rightarrow 1(12 - 8) - 1(6p - 2p^2) = 0$

$\Rightarrow 4 - 6p + 2p^2 = 0$

$\Rightarrow 2(p^2 - 3p + 2) = 0$

$\Rightarrow p^2 - 3p + 2 = 0$

$\Rightarrow p = 1 \text{ or } 2$

Also for these values of $p, \Delta_2, \Delta_3 = 0$

25. (3)

$$\lim_{x\to 2} \frac{(10-x)^{1/3} - 2}{x - 2}$$

$= \lim_{h\to 0} \frac{(8-h)^{1/3} - 2}{h}$ (Put $x = 2 + h$)

$$= \lim_{h\to 0} \frac{2\left(1 - \frac{h}{8}\right)^{1/3} - 2}{h}$$

$$= 2\lim_{h\to 0} \frac{\left(1 - \frac{h}{8}\right)^{1/3} - 1}{h}$$

$$= 2\lim_{h\to 0} \frac{1 - \frac{1}{3}\frac{h}{8} - 1}{h} = -\frac{1}{12}$$

26. Ans. 0016

Sol. Any point on curve $y = x^3$ is of form (t, t^3) and

$$\left.\frac{dy}{dx}\right|_{(t,t^3)} = 3t^2$$

Equation of tangent at (t, t^3) is

$y - t^3 = 3t^2 (x - t)$

$\Rightarrow 3t^2x - y - 2t^3 = 0$ …(i)

The intersection of (i) with $y = x^3$ is given by

$3t^3 x - x^3 - 2t^3 = 0$

$3x (t^2 - x^2) + 2(x^3 - t^3) = 0$

$(x - t) [-3x^2 - 3xt + 2x^2 + 2t^2 + 2xt] = 0$

$(x - t) (-x^2 - xt + 2t^2) = 0$

$x = t$ or $x = 2t$

Therefore, the tangent at $x = t$ meets $y = x^3$ at point P_2

(t_2, t_2^3) when $t_2 = -2t$

The abscissae of $P_1, P_2, \ldots. P_n$ are $t, -2t \ldots (-2)^{n-1} t$(G.P.)

$$\text{Area of } \Delta P_1P_2P_3 = \frac{1}{2} \left| \begin{vmatrix} t & t^3 & 1 \\ -2t & -8t^3 & 1 \\ 4t & 64t^3 & 1 \end{vmatrix} \right|$$

$= \frac{1}{2} t^4 \left| \begin{vmatrix} 1 & 1 & 1 \\ -2 & -8 & 1 \\ 4 & 64 & 1 \end{vmatrix} \right| = \frac{1}{2} t^4 \left| \begin{vmatrix} 1 & 1 & 1 \\ 0 & -6 & 3 \\ 0 & 60 & -3 \end{vmatrix} \right|$

(from $R_2 \to R_2 + 2R_1$, $R_3 \to R_3 - 4R_1$)

$= \frac{1}{2} t^4 \times 162 = 81\ t^4$

area of $\Delta P_2P_3P_4 = 81\ t_2^4$ $\frac{\text{area of } \Delta P_1P_2P_3}{\text{area of } \Delta P_2P_3P_4} = \left(\frac{t}{t_2}\right)^4$

$= \left(\frac{-1}{2}\right)^4 = \frac{1}{16}$

$\therefore \frac{\text{area of } \Delta P_2P_3P_4}{\text{area of } \Delta P_1P_2P_3} = 16$

27. Ans. 0

Sol. Obvious

28. Ans. 0001

Sol. a + b + c = 3

a + b = 3 – c

[a + b] = [3 – c] = 1

$\Rightarrow 1 < c \le 2$

Area A = $\frac{1}{2} \frac{1}{3-c}$

for A maximum 3 – c should be minimum

$A_{max} = \frac{1}{2}$

2A = 1

29. 8

$y = f(t)$ Where $t = \frac{2x+3}{3-2x}$

$\Rightarrow \frac{dy}{dx} = f'(t) \times \frac{dt}{dx} = \sin\left(\log \frac{2x+3}{3-2x}\right) \times \frac{12}{(3-2x)^2}$

$B = 12, A = 2$

$B - 2A = 8$

30. Ans. 0001