PHHYSICS

1. (c)

Energy liberated
$= 2 \times 117 \times 8.5 - 236 \times 7.6$
$= 1989 - 1793.6$
$= 195.4$ MeV ≈ 200 MeV

2. (a)

$$X_C = \frac{1}{2\pi v C} \Rightarrow C = \frac{1}{2\pi v X_C} = \frac{1}{2 \times \pi \times \frac{400}{\pi} \times 25} = 50\ \mu F$$

3. (a)

$$E = Rhc\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$E_{(4\to3)} = Rhc\left[\frac{1}{3^2} - \frac{1}{4^2}\right]$
$= Rhc\left[\frac{7}{9 \times 16}\right] = 0.05\ Rhc$
$E_{(4\to2)} = Rhc\left[\frac{1}{2^2} - \frac{1}{4^2}\right]$
$= Rhc\left[\frac{3}{16}\right] = 0.2\ Rhc$
$E_{(2\to1)} = Rhc\left[\frac{1}{(1)^2} - \frac{1}{(2)^2}\right]$
$= Rhc\left[\frac{3}{4}\right] = 0.75 Rhc$
$E_{(1\to3)} = Rhc\left[\frac{1}{(3)^2} - \frac{1}{(1)^2}\right]$
$= -\frac{8}{9} Rhc = -0.9\ Rhc$
Thus, transition III gives most energy. Transition I represents the absorption of energy.

4. (b)

5. (b)

Equation of wave y=0.2 sin (1.5x+60t)
Comparing with standard equation
y=A sin (kx+ωt)
k=1.5, ω=60
∴ velocity of wave $v = \frac{\omega}{k} = \frac{60}{1.5} = 40 ms^{-1}$
Velocity of wave on a stretched string

$$v = \sqrt{\frac{T}{m}}$$

Where m-linear density
T= tension in the string
So, $T = v^2 m$
$= (40)^2 \times 3 \times 10^{-4} = 0.48$

6. (c)

$Vp^n =$ constant

$\therefore\ Vp^n = \left(V + \frac{\Delta V}{V}\right)\left(1 + n\frac{\Delta p}{p}\right)$

$1 = 1 + \frac{\Delta V}{V} + n\frac{\Delta p}{p} + n\frac{\Delta V}{V}\frac{\Delta P}{p}$

Or $\frac{\Delta V}{V} = -n\frac{\Delta p}{p}$, (neglecting the product)

As $k = \frac{-\Delta p}{\Delta V/V} = \frac{p}{n}$

7. (b)

$B = \frac{\mu_0}{4\pi}\frac{2i_D}{r} = \frac{\mu_0}{4\pi} \times \varepsilon_0 \frac{d\phi_E}{dt}$

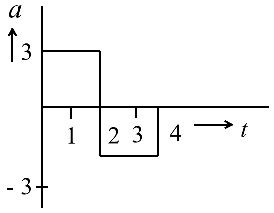
$= \frac{\mu_0}{2\pi}\frac{2i_D}{r} = \frac{\mu_0}{4\pi}\frac{2}{r} \times \varepsilon_0 \frac{d\phi_E}{dt}$

$= \frac{\mu_0\varepsilon_0\pi r^2 dE}{2\pi r dt} = \frac{\mu_0\varepsilon_0 r}{2}\frac{dE}{dt}$

8. (c)

$r = \frac{mv}{qB} \Rightarrow r \propto v$

9. (a)



Takingthe motion from 0 to 2 s

$u = 0, a = 3ms^{-2}, t = 2s, v = ?$

$v = u + at = 0 + 3 \times 2 = 6ms^{-1}$

Taking the motion from 2 s to 4 s

$v = 6 + (-3)(2) = 0ms^{-1}$

10. (a)

In tangent galvanometer experiment the plane of the coil is first set in the magnetic meridian

11. (a)

12. (b)

In the given graph CD represents liquid state

13. (d)

$T = 2\pi\sqrt{\frac{l}{g}}$. T will decrease, If g increases

It is possible when rocket moves up with uniform acceleration

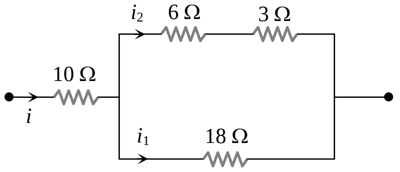
14. (b)

15. (b)

The given circuit can be redrawn as

$\frac{i_1}{i_2} = \frac{9}{18} = \frac{1}{2}$

and $i = i_1 + i_2$



$\Rightarrow \frac{i}{i_1} = 1 + \frac{i_2}{i_1} = 1 + 2 = 3$

From $P = i^2R \Rightarrow \frac{P_{10\Omega}}{P_{18\Omega}} = \left(\frac{i}{i_1}\right)^2 \times \frac{10}{18} \Rightarrow P_{10\Omega} = 10W$

16. (c)

r_p varies with i_p according to relation $r_p \propto r_p^{-1/3}$, $i.e.$, when i_p increases, r_p decreases, hence graph C represents the variation of r_p

μ doesn't depend upon i_p, hence graph A is correct

17. (c)

Acceleration $A = \omega^2 y \Rightarrow \omega = \sqrt{\frac{A}{y}} = \sqrt{\frac{0.5}{0.02}} = 5$

Maximum velocity $v_{\text{max}} = a\omega = 0.1 \times 5 = 0.5$

18. (b)

Young's modulus $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\frac{F}{A}}{\text{Strain}}$

or$Y \frac{mg}{A\times\text{strain}}$

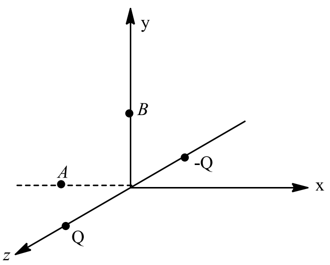
or $m = \frac{Y\times A\times\text{strain}}{\text{g}}$

$= \frac{2\times 10^{11}\times 10^{-3}\times 10^{-6}}{10} = 60\text{kg}$

19. (c)

$A \equiv (-a, 0,0) B \equiv (0, a, 0)$

Point charge is moved from A to B

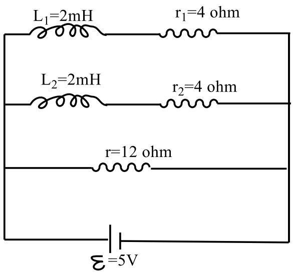


$V_A = V_B = 0$

$\therefore \quad W = 0$

20. (c)

21. (8)



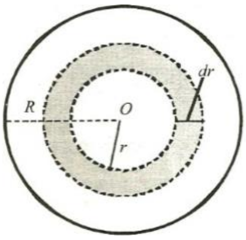
$I_{max} = \frac{\varepsilon}{R} = \frac{5}{12} A$(initially at $t = 0$)

$I_{min} = \frac{\varepsilon}{R_{ep}} = \varepsilon\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{R}\right)$(finally in steady state)

$= 5\left(\frac{1}{3} + \frac{1}{4} + \frac{1}{12}\right) = \frac{10}{3} A$

$\frac{I_{max}}{I_{min}} = 8$

22. (3)



Given $E = ar$

When $r = R, ER = aR$

$\therefore \ \phi = E_R$(area)$= \alpha R 4\pi R^2$

By the Gauss's theorem, the net electric flux is $\frac{1}{\varepsilon_0}$ charge enclosed

$\alpha R 4\pi R^2 = \frac{1}{\varepsilon_0} Q_{\text{enclosed}}$

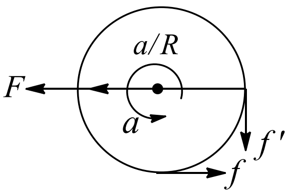
$\therefore \quad Q_{\text{enclosed}} = (4\pi\varepsilon_0)\alpha R^3$

Given $R = 0.30\text{m}, \alpha = 100\text{V/m}^2$

$Q_{\text{enclosed}} = \frac{1}{9 \times 10^9} \times 100 \times (0.30^3) = 3 \times 10^{-10}\text{C}$

23. (4)

Note : If net force applied by the rod is considered to be 2 N



$\sqrt{f^2 + F^2} = 2$ (i)

$FR - f'R = 2mR^2 \frac{a}{R}$

$F - f' = 2ma = 1.2$

From Eqs. (i) and (ii), $(1.2 + f')^2 + f'^2 = 2^2$

$2f'^2 + 2.4 f' + 1.44 = 4$

$f'^2 + 1.2 f' + 0.72 - 2 = 0$

$f'^2 + 1.2 f' - 1.28 = 0$

$f' = \dfrac{-1.2 \pm \sqrt{1.44 + 4 \times (1.28)}}{2}$

$= 0.6 \pm \sqrt{0.36 + 1.28} = -0.6 \pm \sqrt{0.64} = 0.68$

From Eq. (ii), $F = 1.88$

$V = \dfrac{0.68}{1.88} = \dfrac{P}{10} \Rightarrow P = 3.16 \approx 4$

Note: But if only normal reaction applied by the rod is considered to be 2 N

II Law $\Rightarrow 2 - f = 2[0.3] \Rightarrow f = 2 - 0.6$

$f - 1.4\ Nx$ (i)

$A = R\alpha$

$\Rightarrow 0.3 = \alpha[0.5]$

$\Rightarrow \alpha = \frac{3}{5}$ rad/s (ii)

$\tau_c = I_c\alpha \Rightarrow fR - 2\ \mu R = mR^2\alpha$

$f - 2\mu = mR\alpha$

$1.4 - 2\mu = \dfrac{2}{2}\left(\dfrac{3}{2}\right)$

$1.4 - 0.6 = 2\mu$

$0.8 = 2\mu \Rightarrow \mu = 0.4 = \dfrac{P}{10}$

$\therefore P = 4$

24. (1)

Activity $\left(-\frac{dN}{dt}\right) = \lambda N = \left(\frac{1}{t_{mean}}\right) \times N$

$\therefore N = \left(-\frac{dN}{dt}\right) \times t_{mean}$= Total number of atoms

Mass of one radioactive substance

=(number of atoms)×(mass of one atom)

$= \left(-\dfrac{dN}{dt}\right)(t_{mean})(m)$

Substituting the values, we get

Total mass of radioactive substance

=1 mg

∴ Answer is 1.

25. (5)

As the velocity of the ball changes from $\vec{v}_1$ to $\vec{v}_2$, the change in velocity $\Delta\vec{v}$ is given by

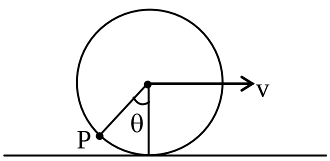
$|\Delta\vec{v}| = \sqrt{v_1^2 + v_2^2 - 2v_1v_2\cos\theta}$

Where $v_1 = 30$ m/s, $v_2 = 40$ m/s and $\theta = 90°$. Then, $|\Delta\vec{v}| = 5$ m/s

$a_{av} = \dfrac{|\Delta\vec{v}|}{\Delta t} = \dfrac{5}{0.01} = 5 \times 10^2 \text{m/s}^2$

26. Ans. 0004

Sol.



$V_P = \sqrt{V^2 + V^2 + 2V^2\cos(\pi - \theta)}$

$\therefore V_P = 2\,V\sin\left(\frac{\theta}{2}\right)$

Now $V_P = \frac{ds}{dt} = \frac{ds}{dv} \cdot \frac{dv}{dt}$

$\therefore V_P = \omega\,\frac{ds}{d\theta} = \frac{V}{R}\,\frac{ds}{d\theta}$

$\therefore \frac{V}{R}\frac{ds}{d\theta} = 2V\sin(\theta/2)$

$\Rightarrow ds = 2R\sin(\theta/2)\,d\theta$

$\therefore S = 2R\int_0^{2\pi}\sin\left(\frac{\theta}{2}\right)d\theta = 8R = 4m$

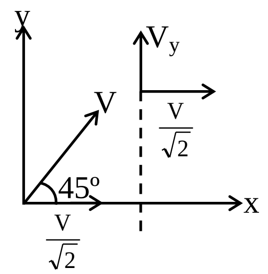
27. Sol.

$N = mg$

$F\,\frac{L}{2}\,\sin\theta = \frac{N^2}{2}\,\cos\theta$

28. **Ans.** 1

Sol.



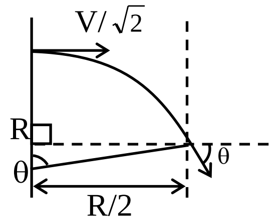
$0.5 = \frac{Vt}{\sqrt{2}},\ t = \frac{\sqrt{2}\times 0.5}{V}$

$V_y = \frac{V}{\sqrt{2}} - \frac{qE}{m}\times t$

$V_y = \frac{V}{\sqrt{2}} - \frac{V^2}{\sqrt{2}V},\ V_y = 0$

$R = \frac{mV}{qB} = \frac{mV}{\sqrt{2}qB}$

$\sin\theta = \frac{R}{2R} = \frac{1}{2},\ \theta = 30^\circ$



Deviation = 45º + 30º = 75º clockwise.

29. Sol. $\frac{\sigma}{2\varepsilon_0}(\pi R^2)$

30. **Ans.** 0060

Sol. $h = ut - \frac{1}{2}gt^2$

or $gt^2 - 2ut + 2h = 0$

$t_1t_2 = \frac{2h}{g}$ and $t_1 + t_2 = \frac{2u}{g} = T$

$\therefore \quad (t_2 - t_1)^2 = (t_1 + t_2)^2 - 4t_1t_2$

$16 = 64 - 4 \times \frac{2h}{g} \Rightarrow h = 60$ m

CHEMISTRY

1. (b)

2. (c)

Meq. of $HNO_3 = 1000 \times 2 = 2000$

$\therefore \frac{w}{63/3} \times 1000 = 2000$

$\therefore w = 42$ g

3. (c)

Larger is the size of atom, lesser is the tendency for overlapping, lesser is bond energy.

4. (d)

5. (c)

Except acetylene, all terminal alkynes have only one acidic H-atom.

6. (b)

As the distance between the atoms, increases, bond polarity increases

7. (a)

Usually across the first transition series, the negative values for standard electrode potential decrease except for Mn due to stable d^5 –configuration.
So, correct order : Mn $> Cr > Fe > Co$

8. (b)

When equal number of cations or anions are missing from their lattice sites (to maintain electrical neutrality), then the defect is called Schottky defect. The defect is observed in highly ionic compounds which have cations and anions of similar size $e.g.$,NaCl, KCl etc.

9. (d)

wt. of 112 litre $O_2 = \frac{32 \times 112}{22400} = 0.16$

10. (b)

Li-Mg shows diagonal relationship due to this fact.

11. (a)

A gas is more soluble if (i) More are forces of attractions among molecules of gases,
ii) More being the tendency of ionization in a solvent and
iii) More is H-bonding .

12. (b)
13. (a)

Linseed oil is commonly used to prepare soap because of low cost.

14. (c)

$\therefore \quad pH = 2$
$\therefore [H^+] = 10^{-2}$
$[H^+] = N.\alpha$
$10^{-2} = 0.1 \times \alpha$
$\alpha = \frac{10^{-2}}{0.1}$
$\alpha = 0.1$

15. (d)

In acid : $H_2O_2 + 2H^+ + 2e \longrightarrow 2H_2O$ (slow)
In alkali : $H_2O_2 + 2e \longrightarrow 2OH^-$(fast)

16. (d)

Grignard reagent give nucleophilic addition (of R^-) at +ve centre.

17. (b)

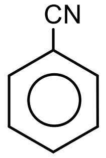
Xe in XeF_4 hassp^3d^2-hybridisation with two lone pair of electrons giving rise to square planar geometry.

18. (c)

Thiokol is a synthetic rubber.

19. (a)

Molecular formula of benzonitrileis C_6H_5CN.



phenyl cyanide or benzonitrile

20. (c)

$PbO \quad + \quad 2HCl \longrightarrow PbCl_2 + H_2O$
Eq. at t = o $\frac{6.5\times2}{224}$ $\frac{3.2}{36.2}$ 00
=0.058 0.088 0 0
Eq. after reaction 0 0.030 0.058 0.058

$\therefore$ Mole of $PbCl_2$ formed $= \frac{0.058}{2} = 0.02$

21. (1)

Rate= $k[a]^n$

Case I: $r_1 = k\left[\frac{a}{V}\right]^n$ ...(i)

Case II: $\frac{r_1}{2} = k\left[\frac{a}{2V}\right]^n$ ...(ii)

By equations (i) and (ii),

$(2)^1 = (2)^n$

$\therefore\ n = 1$

22. (a)

${}_{92}U^{238} + {}_{0}n^1 \rightarrow {}_{54}Xe^{139} + {}_{38}Sr^{94} + {}_{Z}X^A$

Equating mass number on both sides

$235 + 1 = 139 + 94 + A$ or $A = 3$

Equating atomic number on both sides

$92 + 0 = 54 + 38 + Z$ or $Z = 0$

Hence, $x = 3$ neutrons

23. (2)

24. (3)

25. (9)

26. Ans. 0020

Sol. $\Delta G = \Delta H - T\Delta S \Rightarrow \Delta S = 2$ kJ/mol

Additional work = $\Delta G\ (310) - \Delta G\ (300) = \Delta S\ (310 - 300) = 2 \times 10 = 20$ kJ/mol

27. Ans. 3

Sol. $\%R.H = \frac{\text{Partial vapour pressure of water}}{\text{Vapour pressure of water at } 27°C} \times 100$

$\Rightarrow$ Partial vapour pressure of water = $\frac{40}{100} \times 19$

= 7.6 torr

$\Rightarrow$ No. of moles of water vapour in the room

$= \frac{PV}{RT} = \frac{7.6 \times 500 \times 500 \times 500}{760 \times 1000} \times \frac{1}{0.08 \times 300} = 52$

Now using $CaO + H_2O \rightarrow Ca(OH)_2$

wt. of CaO required = $\frac{52 \times 56}{1000}$ = Kg

= 2.912 Kg ~ 3 Kg

28. Ans. 1

Sol. Self Teacher

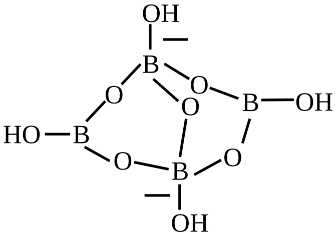
29. Ans. 4

Sol. Self Teacher

30. Ans. 5

Sol. $Na_2B_4O_7$, 10 H_2O exist as $Na_2\ [B_4O_5(OH)_4]$, $8H_2O$

It contain



MATHS

1. (a)

Given, $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{5/2}=\frac{d^3y}{dx^3}$

$$\Rightarrow \left(\frac{d^3y}{dx^3}\right)^2=\left[1+\left(\frac{dy}{dx}\right)^2\right]^5$$

Here, order=3, degree=2

2. (c)

We have,

$f'(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\Rightarrow f(x)'=\lim\limits_{h\to 0}\frac{f(x)+f(h)-f(x)}{h}$

$\Rightarrow f(x)'=\lim\limits_{h\to 0}\frac{f(h)}{h}$

$\Rightarrow f(x)'=\lim\limits_{h\to 0}\frac{h\,g\,(h)}{h}\lim\limits_{h\to 0}g(h)=g(0)\quad [\because g \text{ is conti. at } x=0]$

3. (a)

We have,

$m=$ Slope of the tangent $=-3$

So, the equation of the tangent is

$y=-3x+\left(\frac{2}{-3}\right)\Rightarrow 9x+3y+2=0\ \left[\text{Using}: y=mx+\frac{a}{m}\right]$

4. (b)

The given equation is $x^2-2x\cos\phi+1=0$.

$\therefore x=\frac{2\cos\phi\pm\sqrt{4\cos^2\phi-4}}{2}=\cos\phi\pm i\sin\phi$

Let $\alpha=\cos\phi+i\sin\phi$, then $\beta=\cos\phi-i\sin\phi$

$\therefore\ \alpha^n+\beta^n=(\cos n\phi+i\sin\phi)^n+(\cos\phi-i\sin\phi)^n$

$=2\cos n\phi$

and $\alpha^n\beta^n=(\cos n\phi+i\sin n\phi)(\cos\ n\phi-i\sin n\phi)$

$=\cos^2 n\phi+\sin^2 n\phi=1$

$\therefore$ Required equation is $x^2-2x\cos n\,\phi+1=0$

5. (a)

Let AM be perpendicular from A on BC such that $AM=p$. Then, $BC=4p$. Let $AB=x$ and $AC=y$

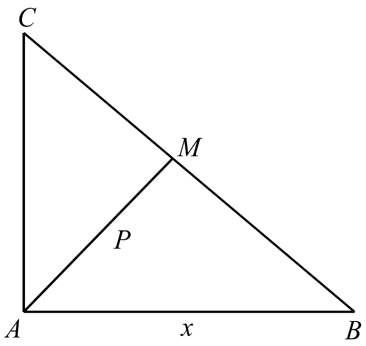
Then,

$\Rightarrow x^2+y^2=(4p)^2$

In ΔABM, we have

$p^2+BM^2=x^2$

$\Rightarrow p^2+(49-k)^2=x^2$, where $k=CM$ ...(i)



In ΔACM, we have

$p^2 = CM^2 = y^2 \Rightarrow p^2 + k^2 = y^2$ ...(ii)

Adding (i) and (ii), we get

$2p^2 + (4p - k)^2 = x^2 + y^2$

$\Rightarrow 2p^2 + (4p - k)^2 = (4p)^2 [\because x^2 + y^2 = (4p)^2]$

$\Rightarrow k = 2p - \sqrt{3}p$

$\Rightarrow BM = BC - CM \Rightarrow BM = 4p - \left(2p - \sqrt{3}p\right) = 2p + \sqrt{3}p$

$\therefore \tan B = \dfrac{AM}{BM} \Rightarrow \tan B = \dfrac{p}{(2+\sqrt{3})p} = 2 - \sqrt{3} \Rightarrow B = 15°$

6. (b)

7. (a)

Given, $\frac{3-|x|}{4-|x|} \geq 0$

$\Rightarrow$ $3 - |x| \leq 0$ and $4 - |x| < 0$

Or $3 - |x| \geq 0$ and $4 - |x| > 0$

$\Rightarrow$ $|x| \geq 3$ and $|x| > 4$

Or $|x| \leq 3$ and $|x| < 4$

$\Rightarrow$ $|x| > 4$ or $|x| \leq 3$

$\Rightarrow (-\infty, -4) \cup [-3, 3] \cup (4, \infty)$

8. (c)

Given that $f(x) = sin\ x - bx + c$

$\therefore f'(x) = \cos x - b$

For decreasing, $f'(x) < 0$, for all $x \in R$.

$\Rightarrow \cos x < b\ for\ all\ x \in R \Rightarrow b > 1$.

9. (c)

$\because -\dfrac{\pi}{2} \leq \sin^{-1} x \leq \dfrac{\pi}{2}, -\dfrac{\pi}{2} \leq \sin^{-1} y \leq \dfrac{\pi}{2}$

And $-\frac{\pi}{2} \leq \sin^{-1} z \leq \frac{\pi}{2}$

Given that, $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$

Which is possible only when

$\sin^{-1} x = \sin^{-1} y = \sin^{-1} z = \dfrac{\pi}{2}$

Or $x = y = z = 1$

Put $p = q = 1$

Then $f(2) = f(1)f(1) = 2 \cdot 2 = 4$

And put $p = 1, q = 2$

Then, $f(3) = f(1)f(2) = 2 \cdot 2^2 = 8$

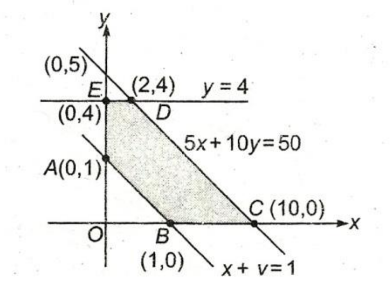
$\therefore x^{f(1)} + y^{f(2)} + z^{f(3)} - \dfrac{x + y + z}{x^{f(1)} + y^{f(2)} + z^{f(3)}}$

$= 1 + 1 + 1 - \dfrac{3}{1 + 1 + 1}$

$= 3 - 1 = 2$

10. (b)

Feasible region is $ABCDEA$ and vertices of the feasible region are $A(0,1), B(1,0), C(10,0), D(2,4)$ and $E(0,4)$



Thus, minimum value of objective function is at (0, 1)

$$\therefore \quad z = 0 \times 2 + 1 = 1$$

11. (c)

Given, $\cos\alpha \cos\beta \cos\gamma = \dfrac{2}{9}$

and $\cos\gamma \cos\alpha = \dfrac{4}{9}$

Then, $\cos\alpha = \dfrac{2}{3}, \cos\beta = \dfrac{1}{3}$ and $\cos\gamma = \dfrac{2}{3}$

$\therefore \cos\alpha + \cos\beta + \cos\gamma = \dfrac{2}{3} + \dfrac{1}{3} + \dfrac{2}{3} = \dfrac{5}{3}$

12. (d)

We have,

$\dfrac{1}{81^n} - \dfrac{10}{81^n}\,{}^{2n}C_1 + \dfrac{10^2}{81^n}\,{}^{2n}C_2 - \dfrac{10^3}{81^n}\,{}^{2n}C_3 + \cdots + \dfrac{10^{2n}}{81^n}$

$= \dfrac{1}{81^n}\{{}^{2n}C_0 - {}^{2n}C_1 10^1 + {}^{2n}C_2 10^2 - {}^{2n}C_3 10^3 + \cdots + {}^{2n}C_{2n} 10^{2n}\}$

$= \dfrac{1}{81^n}(1 - 10)^{2n} = \dfrac{(-9)^{2n}}{81^n} = \dfrac{81^n}{81^n} = 1$

13. (b)

$y = ae^x + be^{-x} + c$

On differentiating w.r.t. x, we get

$y' = ae^x - be^{-x}$

Again differentiating w.r.t. x, we get

$y'' = ae^x + be^{-x}$

Again differentiating w.r.t. x, we get

$y''' = ae^x - be^{-x} = y'$

14. (c)

Given, $A = \begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix}$ of order $n = 2$

$\therefore |\text{adj}(A)| = |A|^{2-1} = \begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix} = 10$

15. (d)

Let $I = \int \frac{\sin x}{\sin(x-\alpha)} dx$

Put $x - \alpha = t \quad \Rightarrow \quad dx = dt$

$\therefore \quad I = \frac{\sin(\alpha + t)}{\sin t} dt$

$= \int \frac{\sin \alpha \cot t + \cos \alpha \sin t}{\sin t} dt$

$= \sin \alpha \int \cot t \, dt + \cos \alpha \int dt$

$= \sin \alpha \log \sin t + + \cos \alpha \,. t + c_1$

$= \sin \alpha \log \sin(x - \alpha) + \cos \alpha \,. (x - \alpha) + c_1$

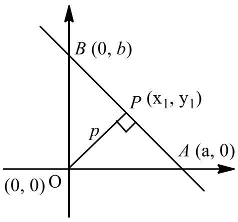
$= \sin \alpha \log \sin(x - \alpha) + x \cos \alpha + c$

[let $c = -\alpha \cos \alpha + c_1$]

16. (b)

Equation of line is $\frac{x}{a} + \frac{y}{b} = 1 \quad \dots$ (i)

Let P be the foot of perpendicular from the origin to the whose coordinate is (x_1, y_1).



Since, $OP \perp AB$

$\therefore$ Slope of $OP \times$ Slope of $AB = -1$

$\Rightarrow \left(\frac{y_1}{x_1}\right)\left(\frac{b}{-a}\right) = -1,$

$by_1 = ax_1 \quad \dots$ (ii)

Since, P lies on the line AB, then

$\frac{x_1}{a} + \frac{y_1}{b} = 1 \;\Rightarrow\; bx_1 + ay_1 = ab \quad \dots$ (iii)

From Eqs. (ii) and (iii), we get

$x_1 = \frac{ab^2}{a^2 + b^2}$ and $y_1 = \frac{a^2 b}{a^2 + b^2}$

Now, $x_1^2 + y_1^2 = \left(\frac{ab^2}{a^2 + b^2}\right)^2 + \left(\frac{a^2 b}{a^2 + b^2}\right)^2$

$\Rightarrow \quad x_1^2 + y_1^2 = \frac{a^2 b^2 (a^2 + b^2)}{(a^2 + b^2)^2}$

$\Rightarrow \quad x_1^2 + y_1^2 = \frac{a^2 b^2}{(a^2 + b^2)}$

$= \dfrac{1}{\dfrac{1}{a^2}+\dfrac{1}{b^2}}$

But $\dfrac{1}{a^2}+\dfrac{1}{b^2}=\dfrac{1}{c^2}$

$\therefore\ x_1^2+y_1^2=c^2$

Thus, the locus of $P(x_1, y_1)$ is

$x^2+y^2=c^2$

Which is the equation of circle.

17. (b)

We know that the mid point of diagonals lies on line $y=2x+c$, here mid point is $(3, 2)$, hence $c=-4$

18. (d)

19. (a)

Probability that at least one shot hits the plane

$= 1-P(\text{none of the shot hits the plane})$

$= 1-0.6\times 0.7\times 0.8\times 0.9$

$= 1-0.3024=0.6976$

20. (b)

Each digit can be placed in 2 ways.

$\therefore$ Required number of ways $=2^{10}$

21. (0)

$$\Delta=\begin{vmatrix}x_1 & y_1 & 0\\ x_2 & y_2 & 0\\ x_3 & y_3 & 0\end{vmatrix}\begin{vmatrix}y_1 & x_1 & 0\\ y_2 & x_2 & 0\\ y_3 & x_3 & 0\end{vmatrix}=0.0=0$$

22. (3)

PLANIf *a*, *b*, *c* are any three vectors

Then$|\vec{a}+\vec{b}+\vec{c}|^2\geq 0$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})\geq 0$

$\therefore \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\geq\dfrac{-1}{2}\left(|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2\right)$

Given, $|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{a}|^2+|\vec{c}-\vec{a}|^2=9$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2-2\,\vec{a}\cdot\vec{b}+|\vec{b}|^2+|\vec{c}|^2-2\,\vec{b}\cdot\vec{c}+|\vec{c}|^2+|\vec{a}|^2$

$-\vec{c}\cdot\vec{a}=9$

$\Rightarrow\quad 6-2\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)=9\ [\because |\vec{a}|=|\vec{b}|=|\vec{c}|=1]$

$\Rightarrow \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=\frac{3}{2}$ ...(i)

Also,$\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\geq\frac{-1}{2}(|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2)$

$\geq -\frac{3}{2}$...(ii)

From Eqs. (i) and (ii),$|\vec{a}+\vec{b}+\vec{c}|=0$

as $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is minimum when $|\vec{a}+\vec{b}+\vec{c}|=0$

$\Rightarrow \quad \vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}} = 0$

$\therefore\ |2\,\mathbf{a} + 5\mathbf{b} + 5\,\mathbf{c}| = |2\,\mathbf{a} + 5\,(\mathbf{b} + \mathbf{c})| = |2\,\mathbf{a} - 5\,\mathbf{a}| = 3$

23. (2)

Let $S = \sum_{r=1}^{\infty} \frac{r+2}{2^{r+1}.r.(r+1)}$

$= \sum_{r=1}^{\infty} \frac{2(r+1) - r}{2^{r+1}.r.(r+1)}$

$= \sum_{r=1}^{\infty} \frac{1}{2^{r+1}} \left(\frac{2}{r} - \frac{1}{r+1}\right)$

$= \sum_{r=1}^{\infty} \left(\frac{1}{2^r.r} - \frac{1}{2^{r+1}(r+1)}\right)$

$= \lim_{n\to\infty} \left[\left(\frac{1}{2^1.1} - \frac{1}{2^2.2}\right) + \left(\frac{1}{2^2.2} - \frac{1}{2^3.3}\right) + \left(\frac{1}{2^3.3} - \frac{1}{2^4.4}\right)\right]$

$= +\ldots + \left(\frac{1}{2^n.n} - \frac{1}{2^{n+1}.(n+1)}\right)$

$= \lim_{n\to\infty} \left(\frac{1}{2} - \frac{1}{2^{n+1}.(n+1)}\right)$

$\therefore\ S = \frac{1}{2}$

Hence, $S^{-1} = 2$

24. (6)

Let $x^2 = 4\cos^2\theta + \sin^2\theta$

Then $(4 - x^2) = 3\sin^2\theta$ and $(x^2 - 1) = 3\cos^2\theta$

$\therefore f(x) = \sqrt{3}|\sin\theta| + \sqrt{3}|\cos\theta|$

$\Rightarrow y_{\min} = \sqrt{3}$ and

$y_{\max} = \sqrt{3}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \sqrt{6}$

Hence range of $f(x)$ is $[\sqrt{3}, \sqrt{6}]$

Hence maximum value of $\left(f(x)\right)^2$ is 6

25. (2)

PLAN $\lim_{x\to 0} \frac{\sin x}{x} = 1$

Given, $\lim_{x\to 1} \left\{\frac{\sin(x-1) + a(1-x)}{(x-1) + \sin(x-1)}\right\}^{\frac{(1+\sqrt{x})(1-\sqrt{x})}{1-\sqrt{x}}} = \frac{1}{4}$

$$\lim_{x\to 0} \left\{\frac{\frac{\sin(x-1)}{(x-1)} - a}{1 + \frac{\sin(x-1)}{(x-1)}}\right\}^{1+\sqrt{x}} = \frac{1}{4}$$

$\Rightarrow \quad \left(\frac{1-a}{2}\right)^{\mathbf{2}} = \frac{1}{4} \quad \Rightarrow \quad (a-1)^2 = 1$

$\Rightarrow \quad a = 2$ or 0

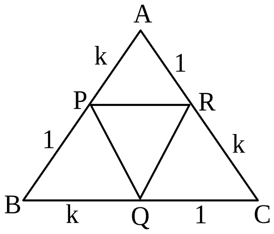
Hence, the maximum value of a is 2.

26. Ans. 2

Sol. Area of ΔARP = ar ΔBPQ = ar ΔQRC

$\therefore$ area of ΔPQR = area ΔABC – 3ar ΔARP

$= \Delta - 3.\ \frac{1}{2}$ AP. AR sin A



$= \Delta - \frac{3}{2}\ \frac{kc}{(k+1)} \cdot \frac{b}{(k+1)} \sin A$

$= \Delta - 3\ \frac{k\Delta}{(k+1)^2} = \frac{(k^2-k+1)\Delta}{(k+1)^2}$

$\{$when $\Delta = \frac{1}{2}$ bc sin A$\}$

$\therefore\ \frac{ar\,\Delta PQR}{ar\,\Delta ABC} = \frac{k^2-k+1}{(k+1)^2} = \frac{1}{3} \Rightarrow k = 2$

27. Ans. 0004

Sol. log ax = 2 log (x + 1)

$\Rightarrow ax = (x+1)^2 \Rightarrow x^2 + (2-a)x + 1 = 0$

Let x_1 and x_2 be roots

$x_1 = \frac{a-2+\sqrt{a^2-4a}}{2}$ and

$x_2 = \frac{a-2-\sqrt{a^2-4a}}{2}$

For solution exists $a^2 - 4a \geq 0$

$a \geq 4$, $a \leq 0$

For positive values $a \geq 4$

For $a > 4$, x_1 and x_2 are different

For $a = 4$, x_1 and x_2 are same

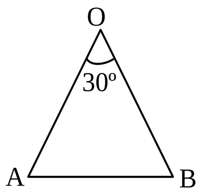
Hence a = 4.

28. Ans. 1

Sol. Self Teacher

29. Ans. 2

Sol. $\frac{OB}{\sin A} = \frac{AB}{\sin 30^o}$



OB = 2AB sin A = 2 sin A

$OB_{max} = 2$

30. Ans. 0026

Sol. Let us take $P(x) = a(x-2)^4 + b(x-2)^3 + c(x-2)^2 + d(x-2) + e$

$-1 = P(2) = e$

$0 = P'(2) = d$

$2 = P''(2) = 2c \Rightarrow c = 1$

$-12 = P'''(2) = 6b \Rightarrow b = -2$

$24 = P^{iv}(2) = 24a \Rightarrow a = 1$

Thus $P''(x) = 12(x-2)^2 - 12(x-2) + 2$

$\Rightarrow P''(1) = 12 - 12(-1) + 2 = 26$