PHYSICS

1. (d)

The magnetic induction at O due to the current in portion AB will be zero because O lies on AB when extended

2. (b)

$0.8 \times 5 = P \times (3 + 5) \Rightarrow P = 0.5\, m$

3. (b)

Pressure at bottom of the lake $= P_0 + h\rho g$

Pressure at half the depth of a lake $= P_0 + \frac{h}{2}\rho g$

According to given condition

$P_0 + \frac{1}{2}h\rho g = \frac{2}{3}(P_0 + h\rho g) \Rightarrow \frac{1}{3}P_0 = \frac{1}{6}h\rho g$

$\Rightarrow h = \frac{2P_0}{\rho g} = \frac{2 \times 10^5}{10^3 \times 10} = 20m$

4. (c)

5. (b)

For stone to be dropped from rising balloon of velocity 29 m/s

$u = -29\, m/s, t = 10 sec$

$\therefore h = -29 \times 10 + \frac{1}{2} \times 9.8 \times 100$

$= -290 + 490 = 200\, m$

6. (a)

In Case of projectile motion at the highest point

$(v)_{\text{vertical}} = 0$ and $(v)_{\text{horizontal}} = v\cos\theta$

The initial linear momentum of the system will be $mv\cos\theta$. Now as force of blasting is internal and force of gravity is vertical

So linear momentum of the system along horizontal is conserved

$p_1 + p_2 = mv\cos\theta$

$m_1 v_1 + m_2 v_2 = mv\cos\theta$

But it is given that $m_1 = m_2 = \frac{m}{2}$ and as one part retraces its path,

$v_1 = -v\cos\theta$

$$\therefore \frac{1}{2}m(-v\cos\theta) + \frac{1}{2}mv^2 = mv\cos\theta$$

or $v_2 = 3v\cos\theta$

7. (d)

$\frac{X - LFP}{UFP - LFP} = \text{constant}$

Where $X =$ Any given temperature on that scale

L. F. P. = Lower fixed point (Freezing point)

U. F. P. = Upper fixed point (Boiling point)

$\frac{W - 39}{239 - 39} = \frac{39 - 0}{100 - 0}$

$\Rightarrow \frac{W - 39}{200} = \frac{39}{100} \Rightarrow W = 78 + 39 \Rightarrow W = 117^oW$

8. (b)

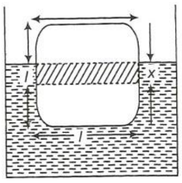
Let at any instant, cube is at a depth x from the equilibrium position then net force acting on the cube = upthrust on the portion of length x

$F = -\rho l^2 x g = -\rho l^2 g x$ ...(i)

Negative sign shows that, force is opposite to x. Hence equation of SHM

$F = -kx$ ...(ii)

Comparing Eqs. (i) and (ii)



$k = \rho l^2 g$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{l^3 d}{\rho l^2 g}} = 2\pi\sqrt{\frac{ld}{\rho g}}$$

9. (b)

$P = VI$

$$I = \frac{550}{220} = 2.5\ A$$

10. (d)

Half-life$T/2 = \frac{T}{1.44} = \frac{100}{1.44}\text{s} = 69.44\text{ s}$

$= \frac{69.44}{60} \approx 1.155\text{ min}$

11. (c)

New potential difference $= \frac{V}{K} = \frac{100}{10} = 10V$

12. (b)

Energy of electron in nth energy level in hydrogen atom

$= \frac{-13.6}{n^2}\text{eV}$

Here, $\frac{-13.6}{n^2} = -3.4\text{ eV}$

So, $n=2$

Angular momentum from Bohr's principle

$= n\frac{h}{2\pi} = \frac{2 \times 6.626 \times 10^{-34}}{2 \times 3.14}$

$= 2.11 \times 10^{-34}\text{Js}$

13. (d)

Now a days microwaves are used to locate the flying objects by radar

14. (b)

$v=\frac{1}{2l}\sqrt{\frac{T}{m}} \Rightarrow \frac{1}{v}=T=2l\sqrt{\frac{m}{T}}$ has the dimensions of time.

15. (d)

$\frac{I}{I_0}=\cos^2\left(\frac{\phi}{2}\right); \phi=\frac{2\pi}{\lambda}.\Delta x$

16. (d)

Breaking force = Breaking stress × Area of cross section of wire
∴ Breaking force ∝ r^2 (Breaking stress is constant)
If radius becomes doubled then breaking force will become 4 times $i.e.$ $40\times 4=160\ kg\ wt$

17. (a)
18. (a)

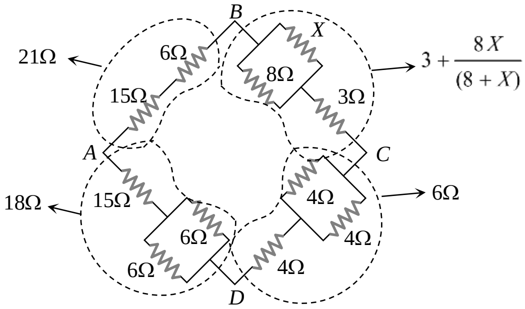
$$\text{Beat frequency}=\frac{\text{Number of beats}}{\text{Time}}=\frac{2}{0.04}=50\ Hz$$

19. (d)

$$v_{rms}=\sqrt{\frac{3RT}{M}} \Rightarrow v_{rms}\propto\frac{1}{\sqrt{M}}$$

20. (c)

Potential difference between B and D is zero, it means Wheatstone bridge is in balanced condition



So $\frac{P}{Q}=\frac{R}{S}\Rightarrow\frac{21}{3+\frac{8X}{(8+X)}}=\frac{18}{6}\Rightarrow X=8\Omega$

21. (5)

$v_x=\frac{dx}{dt}=3$ and $v_y=\frac{dy}{dt}=4-10t=4-10(0)=4$
$v=[v_x^2+v_y^2]^{1/2}=[3^2+4^2]^{1/2}=5\ \text{m/sec}$

22. (6)

Here the charge q_3 attracted towards q_1 and q_2 both, so the net force on q_3 is towards the origin. As only force acting is conservative, we can use conservation of mechanical energy.
Let y = speed of particle at origin.

From energy of conservation principle, we get

$Ui + Ki = U + K$

Or $\frac{1}{4\pi\varepsilon_0}\left[\frac{q_3+q_2}{(r_{32})_i} + \frac{q_3q_1}{(r_{31})_i} + \frac{q_2q_1}{(r_{21})_i}\right] + 0$

$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_3+q_2}{(r_{32})_f} + \frac{q_3q_1}{(r_{31})_f} + \frac{q_2q_1}{(r_{21})_f}\right] + \frac{1}{2}mv^2$

Here, $(r_{21})_i = (r_{21})_f$

Substituting the proper values,

$9 \times 10^9\left[\frac{(-4)(2)}{5} + \frac{(-4)(2)}{3}\right] \times 10^{-12}$

$9 \times 10^9\left[\frac{(-4)(2)}{5} + \frac{(-4)(2)}{3}\right] \times 10^{-12} + \frac{1}{2} \times 10^{-3}v^2$

Or $(9 \times 10^{-3})\left(\frac{-16}{5}\right) = (9 \times 10^{-3})\left(\frac{-16}{3}\right) + \frac{1}{2} \times 10^{-3}v^2$

Or $9 \times 10^{-3}(16)\left(\frac{2}{15}\right) = \frac{1}{2} \times 10^{-3}v^2$

Or $v = 6.2\text{m/s} = 6\text{m/s}$

23. (8)

Potential difference across the coil is $V = L\frac{di}{dt}$

or $V = (1)(4) = 4$ V

Now energy stored per unit time = power

$= Vi = (4)(2) = 8$ J/s

24. (3)

To save himself, the man throws his jacket in opposite direction to the lake According to momentum conservation, he himself gets a velocity in the direction of the lake. During the motion as gravity is the only external force on the system (man plus jacket), centre of mass will not be displaced horizontally. Thus, centre of mass of the system falls vertically and when the man falls in the lake, jacket falls at a point such that the centre of mass of the man and the jacket will be directly below the point from where the man jumps)

As it is given that man falls at a distance d from this point, it implies that jacket will fall at adistance x in the opposite direction such that

$mx = Md \Rightarrow x = \frac{M}{m}d = 29$ m, man has to travel a distance$x + d = 29 + 1 = 30$ m to pick his jacket

25. (4)

Number of Nuclei decayed in time t,

$N_d = N_0(1 - e^{-\lambda t})$

$\therefore$ % decayed=$\left(\frac{N_d}{N_0}\right) \times 100$

$= (1 - e^{-\lambda t}d) \times 100$ ...(i)

Hence, $\lambda = \frac{0.693}{138} = 5 \times 10^{-4}s^{-1}$

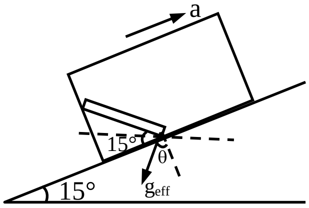
$\therefore$ % decayed $= (\lambda t) \times 100$

$= (5 \times 10^{-4})(80)(100)$

$= 4$

26. Sol. $I\,\omega = \text{const.} \Rightarrow \omega \propto \frac{1}{I}$

$\therefore$ Percentage change in ω
$= -$ (% change in I)
$= -(2 \times \Delta\theta \times 100)$
$= -(2 \times 5 \times 10^{-4} \times -20 \times 100)$
$= 2\ \%$



27. Ans. 4

Sol. $v = \int_0^4 a\,dt = \int_0^2 a\,dt + \int_2^4 a\,dt$

$= \int_0^2 (2-t)dt + \int_2^4 (t-2)dt$

$= 4$ m/s

28. $(40)M_{ice}\,L_f + m_{ice}\,(40-0)\,C_w = m_{steam}L_v + m_{steam}(100-40)\,C_w$

$\Rightarrow 200[80 + 40(a)] = m[540 + 60(a)]$

$\Rightarrow 200\,(120) = m(600)$

$m = 40$ gm

29. Ans.0001

Sol. $R = \dfrac{mv}{qB}$

$q \times 12 \times 10^3 = \dfrac{1}{2}\, m \times (10^6)^2$

$\dfrac{24\times10^3}{10^{12}} = \dfrac{m}{q}$

$R = \dfrac{24\times10^3\times10^6}{10^{12}\times0.2}$

$R = 12 \times 10^{-2}$ m

$R = 12$ cm

30. Sol. $\vec{F}_{ab} = 0, \vec{F}_{bc} = (-0.04N)\hat{i}$

$\vec{F}_{cd} = (-0.04N)\hat{k},\ \vec{F}_{da} = (0.04\hat{i} + 0.04\hat{k})N$

CHEMISTRY

1. (b)

$_{84}Po$ is the only radioactive element of gp 16.

2. (a)

—COOH gp. of salicylic acid is replaced during nitration and halogenation.

3. (d)

BeO is basic oxide and reacts only with an acid to form the salt while
ZnO, SnO_2 and $Al_2O_3$are amphoteric oxides which are react with acid and base both.

4. (b)

$Ag^+ + e \longrightarrow Ag$; finely divided Ag is black in colour and thus.$AgNO_3$ causes black stain on skin. It is therefore, called lunar caustic.

5. (c)

Na reacts with alcohol;

$Na + C_2H_5OH \longrightarrow C_2H_5ONa + \frac{1}{2}H_2$

6. (b)

Due to $+$ve IE in alkyamines and resonance in $C_6H_5NH_2$.

7. (b)

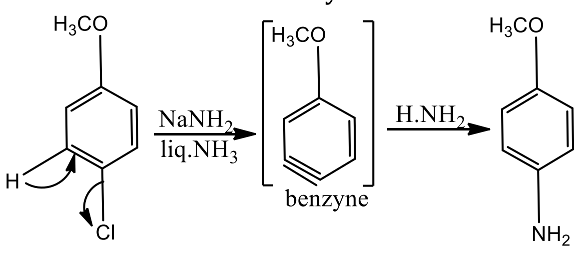
Saran is a copolymer of vinyl chloride and vinyledene chloride.

8. (d)

Mole fraction of $C_6H_6 = \frac{\frac{7.8}{78}}{\frac{7.8}{78}+\frac{46}{92}} = \frac{1}{6}$

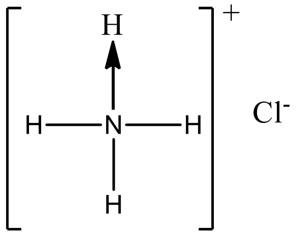
9. (a)

This reaction follows benzyne mechanism.

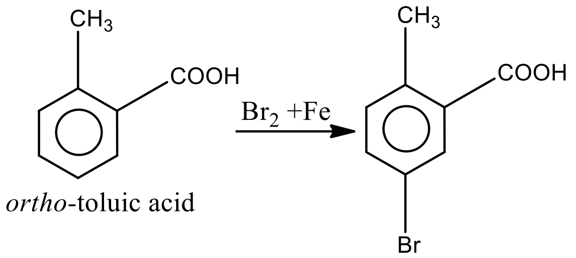


10. (b)

NH_4Clcontains ionic, covalent and coordinate linkage.



11. (c)



($\because$ In the product, $-$Br is*para*to $-$ CH_3 and*meta*to $-$ COOH.)

12. (a)

It is a fact. Rest all are used in pigments.

13. (a)

$Fe^{2+} \longrightarrow Fe^{3+} + e$
$O_2^{1-} + 2e \longrightarrow 2O^{2-}$

14. (b)

At *A* $\rightarrow$ temperature $= T$, volume $= V$, pressure $= p_1$
At *C* $\rightarrow$ temperature $= 2T$ volume $= 2V$, pressure $= p_2$
$$\frac{p_1 V}{T} = \frac{p_2 \times 2V}{2T}$$
$p_1 = p_2$, *ie*,system is isobaric

15. (b)

Hydrolysis of ester catalysed by a proton is acid-base catalysis.

16. (a)

In 12 g carbon, mass of C-14 isotope $= 12 \times \frac{2}{100} = 0.24g$
$\therefore$ Number of C-14 atoms in 12 g of $C = \frac{0.24}{14} \times 6.02 \times 10^{23}$
$= 1.032 \times 10^{22}$

17. (b)

Let the oxidation number of carbonyl carbon in methanal (HCHO) and methanoic acid (HCOOH) is x and y is respectively.
In HCHO,
$2(+1) + x + (-2) = 0$
$2 + x - 2 = 0$

$$x = 0$$

In HCOOH,

$$2(+1) + y + 2(-2) = 0$$

$2+y - 4 = 0$

$$y = 2$$

18. (a)

Amorphous solids neither have ordered arrangement (i.e., no definite shape) nor have sharp melting point like crystals, but when heated they become pliable until they assume the properties usually related to liquids. It is therefore, they are regarded as super cooled liquids.

19. (c)

20. (a)

$\mathrm{pH} = \mathrm{p}K_a + \log\frac{[\text{Conjugate base}]}{[\text{Acid}]}$

$\mathrm{pH} = \mathrm{p}K_a + \log 1$ ($\because$ 50% neutralization)

$\therefore \quad \log \mathrm{H}^+ = -\log 2 \times 10^{-4}$

or $\quad \mathrm{H}^+ = 2 \times 10^{-4}$

21. (4)

22. (6)

Statement (a), (b), and (c) are correct

Hence, total score = $1 + 2 + 3 = 6$

Statement (a):pH = 0, means $[\mathrm{H}^\oplus] = 1$ M

$E^o{}_{\text{red}}$ of $MnO^\ominus{}_4|Mn^{2+} > E^\ominus{}_{\text{red}}$ of $Fe^{3+}|Fe^{2+}$

So $MnO_4{}^\ominus$ will undergo reduction and acts as strong oxidant whereas Fe^{2+} undergoes oxidation. Statement (a) is correct

Statement (b):$MnO_4{}^\ominus$ titrations in the presence of HCl are unsatisfactory since $Cl^\ominus$ is oxidized to Cl_2. Statement (b) is correct

Statement (c): Since $E^\ominus{}_{\text{red }Ce^{4+}|Ce^{3+}} > E^\ominus{}_{\text{red }MnO_4^\ominus|Mn^{2+}}$. So Ce^{4+} will reduce to Ce^{3+}. So $MnO_4{}^\ominus$ cannot oxidize $Ce^{3+}$to Ce^{4+}. Statement (c) is correct

Statement (d):Fe^{2+} can be titrated against $KMnO_4$ in acid medium $([\mathrm{H}^\oplus] = 1\mathrm{M})$

Since $E^\ominus{}_{\text{red }MnO_4^\ominus|Mn^{2+}} > E^\ominus{}_{\text{red }Fe^{3+}|Fe^{2+}}$

So Fe^{2+} can be oxidized to Fe^{3+} by $MnO_4{}^\ominus$

But Ce^{3+} will not be oxidized to Ce^{4+}

Since $E^\ominus{}_{\text{red }Ce^{4+}|Ce^{3+}} > E^\ominus{}_{\text{red }MnO_4^\ominus|Mn^{2+}}$

So, statement (d) is wrong

23. (8)

24. (4)

$t_{1/2} = \frac{0.0693}{k_1}$

Also, $t_{93.75} = \frac{2.303}{k_1}\log\frac{100}{100-93.75}$

$= \frac{2.303}{k_1} \log \frac{100}{6.25}$

$= \frac{2.303}{k_1} \log 2^4$

$= \frac{4 \times 2.303 \times \log 2}{k_1} = \frac{4 \times 0.693}{k_1} = 4t_{1/2}$

25. (5)

26. **Ans**. 1232 millimole MnO_4^-

Sol. Self Teacher

27. **Ans.** 353

Sol. $T_f = k_f \times m$

$\Delta T_f = 1.86 \times 1 = 1.86$

Now, $\Delta T_f = \frac{1000 \times k_f \times w}{m \times W}$

$1.86 = \frac{1000 \times k_f \times w}{342 \times W}$

Or $\frac{w}{W} = \frac{342}{1000} = 0.342$

w + W = 1000

$w_{solute} = 254.84$ gm

$W_{H_2O} = 745.16$ gm.

$\Delta T = \frac{1000 \times 1.86 \times 254.84}{342 \times W_1} = 3.534$

or $W_1 = 392.18$ gm

Therefore amount of ice separated

$= W - W_1 = 745.16 - 392.18$

= 352.98 gm ~ 353 gm.

28. **Ans.**356

Sol.

$\log \frac{p_2}{p_1} = \frac{\Delta H_{vap}}{2.303 \times R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

$\log \frac{p_2}{p_1} = \frac{45.953 \times 10^3}{2.303 \times 8.314}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

$= 2.4 \times 10^3 \times \left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

$p_2 = \frac{1}{2}$ atm, $p_1 = 1$ atm

$\log \frac{1}{2} = 2.4 \times 10^3 \times \left(\frac{1}{T_1} - \frac{1}{T_2}\right) = -\log 2$

$\therefore 2.4 \times 10^3 \times \left(\frac{1}{373} - \frac{1}{T_2}\right) = -0.3$

$\frac{1}{373} - \frac{1}{T_2} = -\frac{0.3}{2.4} \times 10^{-3} = -0.125 \times 10^{-3}$

or $\frac{1}{373} + 0.125 \times 10^{-3} = \frac{1}{T_2}$

or $\frac{1}{T_2} = 2.681 \times 10^{-3} + 0.125 \times 10^{-3}$

$= 2.806 \times 10^{-3}$

or $T_2 = \frac{1000}{2.806} = 356.38$ K ~ 356 K

29. Ans. 0356

Sol. $\log \frac{p_2}{p_1} = \frac{\Delta H_{vap}}{2.303 \times R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

$\log \frac{p_2}{p_1} = \frac{45.953 \times 10^3}{2.303 \times 8.314}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

$= 2.4 \times 10^3 \times \left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

$p_2 = \frac{1}{2}$ atm, $p_1 = 1$ atm

$\log \frac{1}{2} = 2.4 \times 10^3 \times \left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

$= -\log 2$

$\therefore 2.4 \times 10^3 \times \left(\frac{1}{373} - \frac{1}{T_2}\right) = -0.3$

$\frac{1}{373} - \frac{1}{T_2} = -\frac{0.3}{2.4} \times 10^{-3}$

$= -0.125 \times 10^{-3}$

or $\frac{1}{373} + 0.125 \times 10^{-3} = \frac{1}{T_2}$

or $\frac{1}{T_2} = 2.681 \times 10^{-3} + 0.125 \times 10^{-3}$

$= 2.806 \times 10^{-3}$

or $T_2 = \frac{1000}{2.806} = 356.38$ K $\simeq 356$ K

30. Ans. 55

Sol. In vapour phase 1 mole or 78 gm benzene has a volume at 20ºC $= \frac{78 \times 1}{0.877} \times 2750$ ml

1 mole of 92 gm toluene has volume at 20ºC

$= \frac{92 \times 1}{0.867} \times 7720$ ml

$\therefore \frac{p^o_B}{760} \times \frac{78 \times 2750}{0.877 \times 1000} = 1 \times 0.0821 \times 293$

$p^o_B = 74.74$ mm

Similarly, $\frac{P^o_T}{760} \times \frac{92 \times 7720}{0.867 \times 1000}$

$= 1 \times 0.0821 \times 293$

$P^o_T = 22.37$ mm.

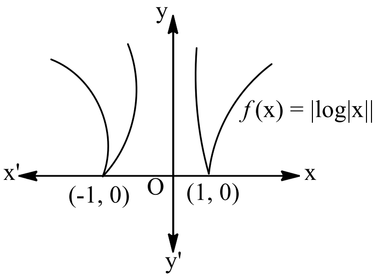
$\therefore 46 = 74.74\, x_B + 22.37\,(1 - x_B)$

or $x_B = 0.55$

or $\%\, x_B = 55$

MATHS

1. (b)



From the graph of $f(x) = |\log|x||$ it is clear that $f(x)$ is everywhere continuous but not differentiable at $x = \pm 1$, due to sharp edge

2. (a)

Given, $\frac{dy}{dx} = \frac{x-2y+1}{2x-4y}$

Put $x - 2y = z \Rightarrow 1 - 2\frac{dy}{dx} = \frac{dz}{dx}$

$\therefore \frac{1}{2}\left[-\frac{dz}{dx} + 1\right] = \frac{z+1}{2z}$

$\Rightarrow zdz = -dx$

$\Rightarrow \frac{z^2}{2} = -x + c_1$ [integrating]

$\Rightarrow (x-2y)^2 + 2x = c$

3. (c)

We have, $\tan^{-1}\frac{x-1}{x+2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left[\frac{\frac{x-1}{x+2} + \frac{x+1}{x+2}}{1 - \left(\frac{x-1}{x+2}\right)\left(\frac{x+1}{x+2}\right)}\right] = \frac{\pi}{4}$

$\Rightarrow \left[\frac{2x(x+2)}{x^2 + 4 + 4x - x^2 + 1}\right] = \tan\frac{\pi}{4}$

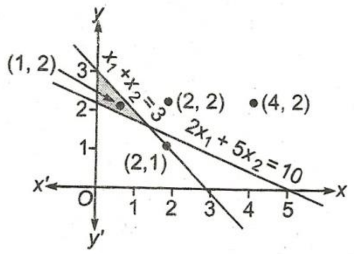
$\Rightarrow \frac{2x(x+2)}{4x+5} = 1$

$\Rightarrow 2x^2 + 4x = 4x + 5$

$\Rightarrow x = \pm\sqrt{\frac{5}{2}}$

4. (b)

It is clear from the figure that points (2, 2), (4, 2) and (2, 1) lies outside the feasible be region and only the point (1, 2) lies in the feasible region



5. (c)

Let α and β are the roots of the given equation

Then, $\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$

Also given, $\alpha+\beta=\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

$$=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha^2\beta^2}$$

$$\Rightarrow \left(-\frac{b}{a}\right)=\left(\frac{-b/a}{c/a}\right)^2-\frac{2}{c/a}$$

$$\Rightarrow -\frac{b}{a}=\left(\frac{b}{c}\right)^2-\frac{2a}{c}$$

$$\Rightarrow \frac{2a}{c}=\frac{b}{c}\left(\frac{b}{c}+\frac{c}{a}\right)$$

$$\Rightarrow \frac{2a}{b}=\frac{b}{c}+\frac{c}{a}$$

$\Rightarrow \frac{c}{a},\frac{a}{b},\frac{b}{c}$ are in AP

$\Rightarrow \frac{a}{c},\frac{b}{a},\frac{c}{b}$ are in HP

6. (c)

The primitive of the given function is

$$\int\frac{dx}{\sqrt{\sin^3 x\sin(x+\theta)}}=\int\frac{dx}{\sqrt{\sin^3 x(\sin x\cos\theta+\cos x\sin\theta)}}$$

$$=\int\frac{\operatorname{cosec}^2 x\,dx}{\sqrt{\cos\theta+\cot x\sin\theta}}$$

Put $\cot x=t\ \Rightarrow -\operatorname{cosec}^2 x\,dx=dt$

$$=-\frac{1}{\sqrt{\sin\theta}}\int\frac{dt}{\sqrt{\cot\theta+1}}$$

$$=-\frac{2}{\sqrt{\sin\theta}}(\cot\theta+t)^{1/2}+c$$

$$=-\frac{2}{\sin\theta}(\cos\theta+t\sin\theta)^{1/2}+c$$

$$=\frac{-2\operatorname{cosec}\theta(\sin x\cos\theta+\sin\theta\cos x)^{1/2}}{\sqrt{\sin x}}+c$$

$$=-2\operatorname{cosec}\theta\left(\frac{\sin(\theta+x)}{\sin x}\right)^{1/2}+c$$

7. (e)

The centres and radii of gives circles are $C_1(0,0), C_2(4,0)$ and $r_1 = 2,\ r_2 = 2$
Now, $C_1C_2 = \sqrt{(4-0)^2 + 0} = 4$
and $r_1 + r_2 = 2 + 2 = 4$
$\therefore\ C_1C_2 = r_1 + r_2$
Hence, three common tangents are possible

8. (a)

In order to remove first degree terms from the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ the origin is shifted at $(-g/a, -f/a)$
In the equation $2x^2 + 7y^2 + 8x - 14y + 4 = 0$, we have
$a = 2, b = 7, g = 4$ and $f = -7$
Hence, the coordinates of the required point are $(-4/2, -7/7) = (-2,1)$

9. (b)

$\because$ Point $P(a,b)$ lies on $3x + 2y = 13$
$\therefore\ 3a + 2b = 13$ ...(i)
and point $Q(b,a)$ is lies on $4x - y = 5$
$\therefore\ 4b - a = 5$ ...(ii)
On solving Eqs. (i) and (ii), we get $a = 3, b = 2$
Therefore, the coordinates of P and Q are (3, 2) and (2, 3)
respectively.
Now, equation of PQ is
$y - 2 = \frac{3-2}{2-3}(x-3) \Rightarrow x + y = 5$

10. (a)

$$\sum_{k=0}^{10} {}^{20}C_k = {}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \cdots + {}^{20}C_{10}$$

On putting $x = 1$and $n = 20$in $(1+x)^n$
$= {}^{n}C_0 + {}^{n}C_1 x + {}^{n}C_2 x^2 + \cdots + {}^{n}C_n x^n$
We get
$2^{20} = 2({}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \cdots + {}^{20}C_9) + {}^{20}C_{10}$
$\Longrightarrow 2^{19} = ({}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \cdots + {}^{20}C_9) + \frac{1}{2}{}^{20}C_{10}$
$\Longrightarrow 2^{19} = {}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \cdots + {}^{20}C_{10} - \frac{1}{2}{}^{20}C_{10}$
$\Longrightarrow {}^{20}C_0 + {}^{20}C_1 + \ldots + {}^{20}C_{10} = 2^{19} + \frac{1}{2}{}^{20}C_{10}$

11. (a)

Given ,
${}^{8}C_r - {}^{7}C_3 = {}^{7}C_2$
$\Rightarrow {}^{8}C_r = {}^{7}C_3 + {}^{7}C_2$
$\Rightarrow\ {}^{8}C_r = {}^{8}C_3$
$\Rightarrow r = 3$

12. (a)

Let the point $P(x, y, z)$ divides the line joining the points A and B in the ratio $m: 1$.

A (5,-3,-2) —m— P —1— (1,2,-2) B

Since, point P is on XOZ-plane
$\therefore$ y coordinate $= 0$
$\Rightarrow \dfrac{2m-3}{m+1} = 0 \Rightarrow m = \dfrac{3}{2}$
Now, $\quad x = \dfrac{3+2\times 5}{3+2} = \dfrac{13}{5}$
and $z = \dfrac{3\times(-2)+2\times(-2)}{5} = -2$
$\therefore$ Required points is $\left(\dfrac{13}{5}, 0, -2\right)$

13. (b)

Given curve is $f(x) = \frac{1}{x+1} - \log(1+x)$
On differentiating w.r.t. x, we get
$f'(x) = -\dfrac{1}{(x+1)^2} - \dfrac{1}{1+x}$
$\Rightarrow \quad f'(x) = -\left[\dfrac{1}{x+1} + \dfrac{1}{(x+1)^2}\right]$
$\Rightarrow \quad f'(x) = -$ve, when $x > 0$
$\therefore \quad f(x)$ is a decreasing function

14. (b)

$\sin^{-1} x + \sin^{-1} y = \dfrac{\pi}{2}$
$\Rightarrow \sin^{-1} x = \dfrac{\pi}{2} - \sin^{-1} y$
$\Rightarrow \sin^{-1} x = \cos^{-1} y$
$\Rightarrow y = \sqrt{1-x^2}$
On differentiating w.r.t. x, we get
$\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{1-x^2}}(-2x) = -\dfrac{x}{y}$

15. (c)
16. (c)

$(\sim p \wedge q) \wedge \sim q = \sim p \wedge (q \wedge \sim q) = \sim p \wedge c = c$

17. (b)

We have,
$C = 60°$
$\Rightarrow \cos C = \dfrac{1}{2} \Rightarrow \dfrac{a^2+b^2-c^2}{2ab} = \dfrac{1}{2} \Rightarrow a^2+b^2-c^2 = ab$ ... (i)
Now,

$\frac{a}{b+c}+\frac{b}{c+a}$

$=\frac{ac+a^2+b^2+bc}{bc+ab+c^2+ac}=\frac{c^2+ac+bc+ab}{c^2+ac+bc+ab}=1$ [Using : (i)]

18. (a)

If $f(x)>0$,then $D<0$

$4a^2-4(10-3a)<0$

$\Rightarrow\ (a+5)(a-2)<0$

$\Rightarrow\ -5<a<2$

19. (c)

Matrices $A+B$ and AB are defined only if both A and B are of same order $n\times n$.

20. (a)

Since, A and B are mutually exclusive events, therefore

$A\cap B=\phi\Rightarrow A\subseteq\bar{B}$ and $B\subseteq\bar{A}$

$\Rightarrow\ P(A)\le P(\bar{B})$and $P(B)\le P(\bar{A})$

21. (0)

$\begin{vmatrix}3u^2 & 2u^3 & 1\\ 3v^2 & 2v^3 & 1\\ 3w^2 & 2w^3 & 1\end{vmatrix}=0$

$R_1\to R_1-R_2$and $R_2\to R_2-R_3$

$\Rightarrow\begin{vmatrix}u^2-v^2 & u^3-v^3 & 0\\ v^2-w^2 & v^3-w^3 & 0\\ w^2 & w^3 & 1\end{vmatrix}=0$

$\Rightarrow\begin{vmatrix}u+v & u^2+v^2+vu & 0\\ v+w & v^2+w^2+vw & 0\\ w^2 & w^3 & 1\end{vmatrix}=0$

$R_1\to R_1-R_2$

$\Rightarrow\begin{vmatrix}u-w & (u^2-w^2)+v(u-w) & 0\\ v+w & v^2+w^2+vw & 0\\ w^2 & w^3 & 1\end{vmatrix}=0$

$\Rightarrow\begin{vmatrix}1 & u+w+v & 0\\ v+w & v^2+w^2+vw & 0\\ w^2 & w^3 & 1\end{vmatrix}=0$

$\Rightarrow(v^2+w^2+vw)-(v+w)[(v+w)+u]=0$

$\Rightarrow v^2+w^2+vw-(v+w)^2-u(v+w)=0$

$\Rightarrow uv+vw+wu=0$

22. (7)

Here, $\lim_{x\to1}\frac{F(x)}{G(x)}=\frac{1}{14}$

$\Rightarrow\ \lim_{x\to1}\frac{F'(x)}{G'(x)}=\frac{1}{14}$ [using L'Hospital's rule] ...(i)

As $F(x)=\int_{-1}^{x}f(t)dt\ \Rightarrow F'(x)=f(x)$ ...(ii)

and $G(x)=\int_{-1}^{x}t|f\{f(t)\}|dt$

$\Rightarrow\ G'(x)=x|f\{f(x)\}|$ ...(iii)

$$\therefore \lim_{x\to 1}\frac{F(x)}{G(x)} = \lim_{x\to 1}\frac{F'(x)}{G'(x)} = \lim_{x\to 1}\frac{f(x)}{x|f\{f(x)\}|}$$

$$= \frac{f(1)}{1\,|f\{f(1)\}|} = \frac{1/2}{|f(1/2)|} \quad \text{...(iv)}$$

Given, $\lim_{x\to 1}\frac{F(x)}{G(x)} = \frac{1}{14}$

$\therefore \frac{\frac{1}{2}}{\left|f\left(\frac{1}{2}\right)\right|} = \frac{1}{14} \quad \Rightarrow \quad \left|f\left(\frac{1}{2}\right)\right| = 7$

23. (7)

$ax^2 + (a+d)x + (a+2d) = 0$
$a, a+d, a+2d$ are in increasing A.P. $(d > 0)$
For real roots $D \geq 0$
$\Rightarrow (a+d)^2 - 4a(a+2d) \geq 0$
$\Rightarrow a^2 - 3a^2 - 6ad \geq 0$
$\Rightarrow (d-3a)^2 - 12a^2 \geq 0$
$\Rightarrow (d-3a)^2 - 12a^2 \geq 0$
$\Rightarrow \left(d - 3a - \sqrt{12}a\right)\left(d - 3a + \sqrt{12}a\right) \geq 0$
$\Rightarrow \left[\frac{d}{a} - \left(3 + 2\sqrt{3}\right)\right]\left[\frac{d}{a} - \left(3 - 2\sqrt{3}\right)\right] \geq 0$
$\therefore \left.\frac{d}{a}\right|_{\text{Min}} = 3 + 2\sqrt{3}$
$\Rightarrow$ least integral value $= 7$

24. (6)

Let $x^2 = 4\cos^2\theta + \sin^2\theta$
Then $(4 - x^2) = 3\sin^2\theta$ and $(x^2 - 1) = 3\cos^2\theta$
$\therefore f(x) = \sqrt{3}|\sin\theta| + \sqrt{3}|\cos\theta|$
$\Rightarrow y_{\min} = \sqrt{3}$ and
$y_{\max} = \sqrt{3}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \sqrt{6}$
Hence range of $f(x)$ is $\left[\sqrt{3}, \sqrt{6}\right]$
Hence maximum value of $\left(f(x)\right)^2$ is 6

25. (1)

$\vec{a}\cdot\vec{b} \Rightarrow \vec{a} \perp \vec{b}$
$\vec{a}\cdot\vec{c} = 0 \Rightarrow \vec{a} \perp \vec{c}$
$\Rightarrow \vec{a} \perp \vec{b} - \vec{c}$
$\left|\vec{a}\times\vec{b} - \vec{a}\times\vec{c}\right| = |\vec{a}\times(\vec{b}-\vec{c})|$
$= |\vec{a}|\left|\vec{b}-\vec{c}\right| = \left|\vec{b}-\vec{c}\right|$
Now $\left|\vec{b}-\vec{c}\right|^2 = \left|\vec{b}\right|^2 + |\vec{c}|^2 - 2\left|\vec{b}\right||\vec{c}|\cos\frac{\pi}{3}$
$= 2 - 2x \times \frac{1}{2} = 1$
$\left|\vec{b}-\vec{c}\right| = 1$

26. 2

Solving $3x + 4y = 9$, $y = mx + 1$ we get $x = \frac{5}{3+4m}$
x is an integer if $3 + 4m = 1, -1, 5, -5$
$\therefore m = \frac{-2}{4}, \frac{-4}{4}, \frac{2}{4}, \frac{-8}{4}$. So, m has two integral values.

27. Ans. 1

Sol. $y = 1/x \Rightarrow \frac{dy}{dx} = \frac{-1}{x^2} \Rightarrow x^2dy + dx = 0$

$$\Rightarrow \frac{x^2}{\sqrt{1+x^4}} dy + \frac{dx}{\sqrt{1+x^4}} = 0$$

$$\Rightarrow \frac{dy}{\sqrt{\frac{1}{x^4}+1}} + \frac{dx}{\sqrt{1+x^4}} = 0$$

$$\Rightarrow \frac{dy}{\sqrt{1+y^4}} + \frac{dx}{\sqrt{1+x^4}} = 0$$

$$\frac{dy}{\sqrt{1+y^4}} + \frac{dx}{\sqrt{1+x^4}} + 1 = 1$$

28. Ans. 2

Sol. Let $A = \text{cosec}^{-1}\left[3x^2 + 1 + \frac{1}{4}\right] + \sec^{-1}\left[3x^2 + \frac{1}{4}\right]$

$= \text{cosec}^{-1}\left(1 + \left[3x^2 + \frac{1}{4}\right]\right) + \sec^{-1}\left[3x^2 + \frac{1}{4}\right]$

where $3x^2 + \frac{1}{4} \geq 1$

Now A will be minimum when $\left[3x^2 + \frac{1}{4}\right]$ is minimum

$\Rightarrow \left[3x^2 + \frac{1}{4}\right] = 1$

$\therefore A_{min} = \text{cosec}^{-1} 2 + \sec^{-1} 1 = \frac{\pi}{6}$

hence $\frac{12}{\pi} A = 2$

29. Ans. 0

Sol. $\because \cot^4x - 2(1 + \cot^2x) + a^2 = 0$
$\cot^4x - 2\cot^2x + a^2 - 2 = 0$
$(\cot^2x - 1)^2 = 3 - a^2$
to have at least one solution $3 - a^2 \geq 0$
$a \in [-\sqrt{3}, \sqrt{3}]$
$\therefore$ integer values of $a = -1, 0, 1$
$\therefore$ sum = 0

30. Ans. 5052