PHYSICS

1. (d)

Control rods or safety rods used in a nuclear reactor are cadmium rods or boron rods

2. (d)

Electric field $E=\frac{V}{l}-\frac{iR}{l}$ $(R=\text{Resistance of wire})$

Magnetic field at the surface of wire $B=\frac{\mu_0 i}{2\pi a}$ $(a=\text{radius of wire})$

Hence poynting vector, directed radially inward is given

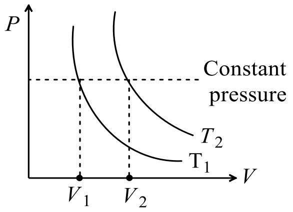
By $S=\frac{EB}{\mu_0}=\frac{iR}{\mu_0 l}.\frac{\mu_0 i}{2\pi a}=\frac{i^2R}{2\pi al}$

3. (b)

Volume $=(2.1\times10^{-2})^3\text{m}^3=9.261\times10^{-6}\text{m}^3$. Rounding off two significant figures, we get $9.3\times10^{-6}\text{m}^3$.

4. (c)

For a given pressure, volume will be more if temperature is more [Charle's law]



From the graph it is clear that $V_2>V_1\Rightarrow T_2>T_1$

5. (c)

Momentum of one piece $=\frac{M}{4}\times3$

Momentum of the other piece $=\frac{M}{4}\times4$

$\therefore$ Resultant momentum $=\sqrt{\frac{9M^2}{16}+M^2}=\frac{5M}{4}$

The third piece should also have the same momentum

Let its velocity be v, then

$\frac{5M}{4}=\frac{M}{2}\times v\ \Rightarrow v=\frac{5}{2}=2.5\ m/sec$

6. (a)

In L-R circuit, the growing current at time t is given y $i=i_0\left[1-e^{-\frac{t}{\tau}}\right]$ where $i_0=\frac{E}{R}$ $and\ \tau=\frac{L}{R}$

$\therefore$ Charge passed through the battery in one time constant is

$$q=\int_0^{\tau} idt=\int_0^{\tau} i_0\left(1-e^{-t/r}\right)dt$$

$q=\ i_0\tau-\left[\frac{i_0e^{-t}}{-2/\tau}\right]_0^t=i_0\tau+i_0\tau[e^{-1}-1]$

$=i_0\tau-i_0\tau+\frac{i_0\tau}{e}$

$q\ =\frac{i_0\tau}{e}=\frac{(E/R)(L/R)}{e}=\frac{el}{eR^2}$

7. (d)

During adiabatic expansion

$TV^{\gamma-1}=\text{constant of }T_2V_2^{\gamma-1}=T_1V_1^{\gamma-1}$

$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$

For monoatomic gas, $\gamma = 5/3$

$\frac{T_1}{T_2} = \left(\frac{AL_2}{AL_1}\right)^{5/3-1} = \left(\frac{L_2}{L_1}\right)^{2/3}$

8. (a)

Spring constant of each part, $k' = 2k$

Original frequency of system

$\alpha = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

where$m$ is the mass of the block.

New frequency of system

$\alpha' = \frac{1}{2\pi}\sqrt{\frac{k'}{m}}$

or $\alpha' = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

$\therefore \alpha' = \sqrt{2}\alpha$

9. (a)

10. (b)

Force constant, $\text{K} = \tan 30° = 1/\sqrt{3}$

11. (a)

Here $v_1 = \sqrt{2\text{g}(h/2)} = \sqrt{gh}$ ...(i)

Using Bernoulli's theorem, we have

$p_a + \rho gh + 2pg(h/2) = p_a + \frac{1}{2}(2\rho)v_2^2$

Or $v_2 = \sqrt{2gh}$ ...(ii)

$\therefore \frac{v_1}{v_2} = \frac{1}{\sqrt{2}}$

12. (b)

Wave number is the reciprocal of wavelength and is written as $\bar{n} = \frac{1}{\lambda}$

13. (d)

$R_1 = \rho\frac{l_1}{A_1}$ and $R_2 = \rho\frac{l_2}{A_2} \Rightarrow \frac{R_1}{R_2} = \frac{l_1}{l_2}.\frac{A_2}{A_1} = \frac{l_1}{l_2}\left(\frac{r_2}{r_1}\right)^2$

Given$\frac{l_1}{l_2} = \frac{1}{2}$ and $\frac{r_1}{r_2} = \frac{2}{1}$or $\frac{r_2}{r_1} = \frac{1}{2} \Rightarrow \frac{R_1}{R_2} = \frac{1}{8}$

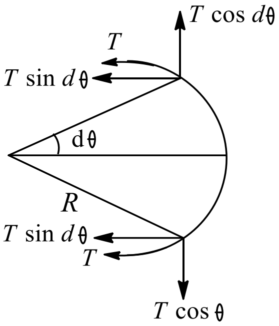
$\therefore$Ratio of heats $\frac{H_1}{H_2} = \frac{V^2/R_1}{V^2/R_2} = \frac{R_2}{R_1} = \frac{8}{1}$

14. (c)

Voltage gain $A_v = \frac{\mu}{1+\frac{r_p}{R_L}}$, for $r_p = R_L \Rightarrow A_v = \frac{\mu}{2}$

15. (a)

For small element proton



$2T \sin d\theta = 2R\, d\theta$ iB

$2Td\theta \ = 2Ribd\theta$

$T \ = iRB$

16. (c)

These photons will be emitted when electron makes transitions in the shown way.
So, these transitions is possible from two or three atoms.
From three atoms separately.

17. (c)

Total distance to be covered for crossing the bridge
= length of train + length of bridge
$= 150m + 850m = 1000m$
Time $= \frac{\text{Distance}}{\text{Velocity}} = \frac{1000}{45\times\frac{5}{18}} = 80\ sec$

18. (a)

The materials for a permanent magnet should have high retentivity (so that the magnet is strong) and high coercivity (so that the magnetism is not wiped out by stray magnetic fields). As the material in this case is never put to cyclic changes of magnetization, hence hysteresis is immaterial.

19. (c)

As is clear from figure.

$$\frac{dQ}{dt} = \frac{dQ_1}{dt} + \frac{dQ_2}{dt}$$

$$\frac{K(A_1 + A_2)dT}{dx} = K_1A_1\frac{dT}{dx} + K_2A_2\frac{dT}{dx}$$

$$K = \frac{K_1A_1 + K_2A_2}{A_1 + A_2}$$

20. (d)

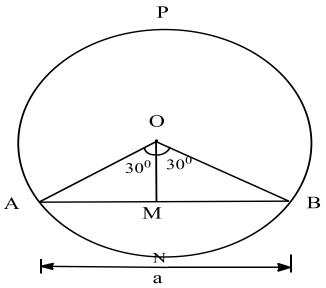
Extra charge $Q = (2CV - CV) = CV$ flows through potential V of the battery. Thus $W = QV = CV^2$

21. (3)

Given $\lambda \times 200 \text{ nm} = 2 \times 10^{-7}\text{m}$
Energy of one photon is
$$\frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2 \times 10^{-7}} = 9.945 \times 10^{-19}$$
Number of photons is
$$\frac{1 \times 10^{-7}}{9.945 \times 10^{-19}} = 1 \times 10^{11}$$
Hence, number of photoelectrons emitted is
$$\frac{1 \times 10^{11}}{10^4} = 1 \times 10^7$$
Net amount of +ve change 'q' developed due to the outgoing electrons= $1 \times 10^7 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-12}\text{C}$
Now potential developed at the centre as well as at the surface due to these charges is
$$\frac{Kq}{r} = \frac{9 \times 10^9 \times 1.6 \times 10^{-12}}{4.8 \times 10^{-2}} = 3 \times 10^{-1}\text{V} = 0.3 \text{ V}$$

22. (6)

$ANBP$is cross-section of a cylinder of length L. The line charge passes through the centre O and perpendicular to paper.



$$AM = \frac{a}{2}, MO = \frac{\sqrt{3}a}{2}$$
$$\therefore \angle AOM = \tan^{-1}\left(\frac{AM}{OM}\right)$$
$$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$$
Electric flux passing from the whole cylinder
$$\phi_1 = \frac{q_{\text{in}}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$
$\therefore$Electric flux passing through $ABCD$ plane surface (shown only AB)= Electric flux passing through cylindrical surface ANB
$$= \left(\frac{60°}{360°}\right)(\phi_1)$$
$$= \frac{\lambda L}{6\varepsilon_0}$$
$$\therefore n = 6$$

23. (3)

For circular motion of the stone,
$\frac{mv^2}{r} = T$ [as g = 0] (i)
and as A.M. is constant,

$mvr = K$,

i.e., $v = \frac{K}{mr}$(ii)

Eliminating v between Eqs. (i) and (ii), we get

$$\frac{m}{r}\left[\frac{K}{mr}\right]^2 = T,$$

$$T = \frac{K_2}{m_2}r^{-3}$$

Or $T = Ar^{-3}$ with $A = \left(\frac{K^2}{m}\right)$(iii)

Comparing Eqs.(iii) with $T = A/r^n$, we find $n = 3$

24. (8)

The mutual inductance of solenoid coil system

$M = \mu_0 n_1 N_2 A_2 = \mu_0 n_1 N_2 \pi r_2^2$

$= 4\pi \times 10^{-7} \times 2 \times 10^4 \times 100 \times \pi \times (0.01)^2 = 8\pi^2 \times 10^{-5}$ H

EMF induced in the coil: $e_2 = -M\frac{\Delta i}{\Delta t}$

$= -8\pi^2 \times 10^{-5} \times \left(\frac{-2-2}{0.05}\right) = 640\,\pi^2 \times 10^{-5}$ V

Required charge:

$$q = i\,\Delta t = \frac{e}{R}\,\Delta t = \frac{640 \times \pi^2 \times 10^{-5}}{40\,\pi^2} \times 0.05 = 8\,\mu\text{C}$$

25. (5)

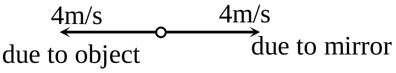
As the velocity of the ball changes from $\vec{v}_1$ to $\vec{v}_2$, the change in velocity $\Delta\vec{v}$ is given by

$$|\Delta\vec{v}| = \sqrt{v_1^2 + v_2^2 - 2v_1 v_2 \cos\theta}$$

Where $v_1 = 30$ m/s, $v_2 = 40$ m/s and $\theta = 90°$. Then, $|\Delta\vec{v}| = 5$ m/s

$$a_{av} = \frac{|\Delta\vec{v}|}{\Delta t} = \frac{5}{0.01} = 5 \times 10^2 \text{m/s}^2$$

26. Ans. 4

Sol.


27. Ans. 1

Sol. (Given |A| = 3)

$|\vec{A}| - |\vec{B}| = 2$

$|\vec{B}| > |\vec{A}|$

$|\vec{B}| = 1$

28. Ans. 4

Sol. Let a = side of cube

p = impulse imparted

∴ After hitting,

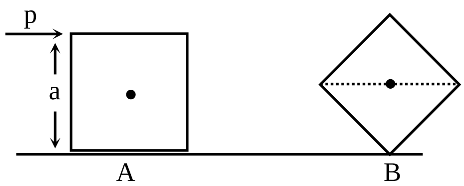
$v_0 = \frac{p}{m}$ and $\omega_0 = \frac{pa}{2I}$

I : moment of inertial about axis passing through C.M.

For just toppling

$\frac{1}{2} I \omega_0^2 = mg\, a\left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right)$

(Applying energy conservation between situation A and B)



$\Rightarrow p = \frac{2I\omega_0}{a} = 4$ kg m/s.

29. $(60) V = 2\alpha_2 + \alpha_1$

$= 10 \times 10^{-6} + 5 \times 10^{-5}$

$= 60 \times 10^{-6}$ /°C

30. **Ans.** 2

Sol. $\vec{v}_1 . \vec{v}_2 = 0 \Rightarrow t = 2 sec$

CHEMISTRY

1. (c)

2. **(a)**

Particle pressure of oxygen $= \frac{2}{1+4+2} \times 2660$

$= 760$ mm

Thus, 1 L oxygen gas is present at $0°C$ and 760 mm pressure.

$\therefore$ Number of oxygen molecules $= \frac{6.023 \times 10^{23}}{22.4}$

3. **(c)**

The adsorption of a gas is directly proportional to the pressure of the gas.

4. **(a)**

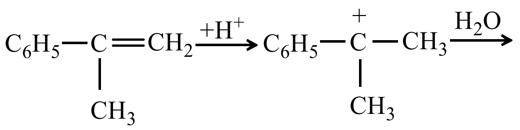
Iodoform test is given by those compounds which have – CH_3CO group or on oxidation yields this group. HCHO does not give this test.

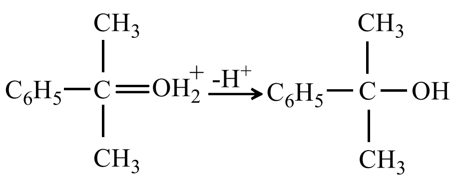
5. **(b)**

$HgS + 2HCl + \underset{(KClO_3)}{3[O]} \longrightarrow HgCl_2 + H_2O + SO_2$

6. **(c)**

The reaction proceeds via carbocation mechanism.





7. (d)

Each possess unpaired electrons.

8. (d)

$Na_2S_2O_3 + 2HCl \longrightarrow 2NaCl + SO_2 + S + H_2O$
$SO_2 + Br_2 + 2H_2O \longrightarrow H_2SO_4 + 2HBr$

9. (b)

Outer shell electrons are referred as valence electrons.

10. (c)

Reaction of aniline with benzaldehyde is condensation reaction.

$$\underset{\text{Benzaldehyde}}{C_6H_5-\overset{\displaystyle H}{\overset{|}{C}}=O} + \underset{\text{aniline}}{H_2NC_6H_5} \xrightarrow{\text{Condensation}} \underset{\text{benzylidene aniline}}{C_6H_5CH=NC_6H_5} + H_2O$$

11. (a)

$$K = 9 = \frac{a \times 10}{(0.1 - a) \times 10}$$

Where a is the molarity of organic compound in CCl_4 at equilibrium

$\therefore \quad a = 0.09\ M$

Thus, molarity of organic compound left in water

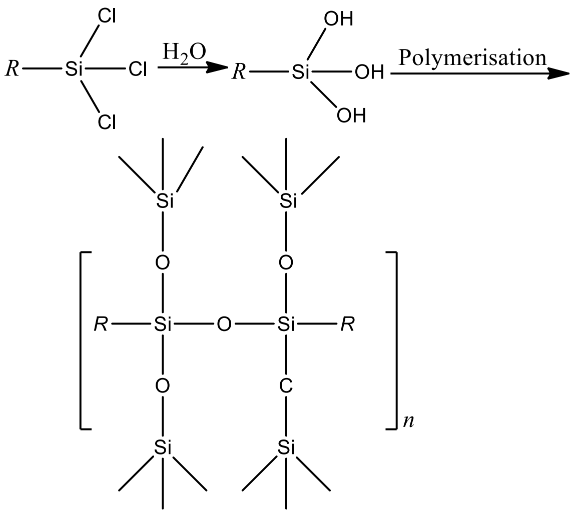
$= 0.1 - 0.09$

$= 0.01\ M$

12. (a)

13. (b)

$R SiCl_3$ gives cross linked silicon polymer on hydrolysis.



14. (c)

$pV = \frac{w}{M}RT$

$M = \frac{wRT}{pV}$

$= \frac{0.455 \times 0.0821 \times 300 \times 760 \times 1000}{800 \times 380}$

$= 28.0$ g

15. (a)

Hydrides are binary compounds of hydrogen. These can be classified in four groups *viz :*
(i) Ionic hydrides *e.g.,* NaH, CaH_2, LiH etc.
(ii) Covalent hydrides *e.g.,* B_2H_6, NH_3, SbH_3 etc.
(iii) Polynuclear hydrides *e.g.,* $LiAlH_4$, $NaBH_4$ etc.
(iv) Interstitial hydrides, in which hydrogen is trapped in the interstial spaces of transition metals.

16. (d)

Ionisation potential is the amount of energy required to take out most loosely bonded electron from isolated gaseous atom. Its value increases in a period. Element having stable configuration have exceptionally high ionisation potential
N has highest ionisation potential among
C, B, O and N ($\because$ N has $2p^3$stable configuration).

17. (c)

According to Le-Chatelier's principle when a system at equilibrium is subjected to change in pressure, temperature or concentration then the equilibrium is disturbed and shifts in a direction where the effect of change is annuled.

$$\text{Ice} \rightleftharpoons \text{water}$$

When pressure is increased in this system, the melting point of ice is decreased $i.e.$, more ice melts and more water is formed.

18. (b)

$SnCl_2 + 2HgCl_2 \longrightarrow Hg_2Cl_2 + SnCl_4$

19. (a)

$CH_2Cl_2 \xrightarrow{HOH} HCHO$

20. (a)

Bones contain $Ca_3(PO_4)_2$.

21. (6)
22. (6)

PLAN This problem is based on concept of quantum number. Follow the following steps to solve this problem. Write all possible orbitals having combination of same principal, azimuthal, magnetic and spin quantum number.
Then count the all possible electrons having given set of quantum numbers.
For $n = 4$, the total number of possible orbitals are

4s	4p	4d	4f
0	-1 0 +1	-2 -1 0 +1 +2	-3 -2 -1 0 +1 +2 +3

According to question $|m_1| = 1$, i.e. there are two possible values of m_l, i.e. +1 and -1 and one orbital can contain maximum two electrons one having $s = +\frac{1}{2}$ and other having
$s = -1/2$.
So, total number of orbitals having $\{|m_l| = 1\} = 6$
Total number of electrons having
$\left\{|m_l| = 1 \text{and } m_s = -\frac{1}{2}\right\} = 6$

23. (5)
24. (6)
25. (3)
26. Ans. 2.65×10^{-19} J
27. Ans. 1
Sol. Mole = molarity × volume = 0.2 × 5 = 1

28. Ans. 400

29. Ans. 1

30. Ans. 1280

MATHS

1. (a)
2. (c)

Given, $\frac{dy}{dx} - \frac{2}{x} y = x^2 e^x$

$\therefore$ IF $= e^{-\int \frac{2}{x} dx} = e^{-\log x^2} = \frac{1}{x^2}$

$\therefore$ Complete solution is $\frac{y}{x^2} = \int \frac{x^2 e^x}{x^2} dx + c$

$\Rightarrow \frac{y}{x^2} = e^x + c$

$\Rightarrow y = x^2(e^x + c)$

When $y = 0, x = 1$, then $c = -e$

$\therefore y = x^2(e^x - e)$

3. (a)

Let $y = m_1 x$ and $y = m_2 x$ be a pair of conjugate diameters of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and let $P(a\cos\theta, b\sin\theta)$ and $Q(a\cos\phi, b\sin\phi)$ be ends of these two diameters. Then,

$m_1 m_2 = -\frac{b^2}{a^2}$

$\Rightarrow \frac{b\sin\theta - 0}{a\cos\theta - 0} \times \frac{b\sin\phi - 0}{a\cos\phi - 0} = -\frac{b^2}{a^2}$

$\Rightarrow \sin\theta\sin\phi = -\cos\theta\cos\phi$

$\Rightarrow \cos(\theta - \phi) = 0$

$\Rightarrow \theta - \phi = \pm\frac{\pi}{2}$

4. (c)

$\therefore \tan^{-1} x - \tan^{-1} y = 0 \Rightarrow x = y$

Also, $\cos^{-1} x + \cos^{-1} y = \frac{\pi}{2} \Rightarrow 2\cos^{-1} x = \frac{\pi}{2}$

$\Rightarrow \cos^{-1} x = \frac{\pi}{4} \Rightarrow x = \frac{1}{\sqrt{2}} \Rightarrow x^2 = \frac{1}{2}$

Hence, $x^2 + xy + y^2 = 3x^2 = \frac{3}{2}$

5. (b)

Let $y = \sqrt{\frac{1-\sin 2x}{1+\sin 2x}} = \frac{\cos x - \sin x}{\cos x + \sin x}$

$= \frac{1 - \tan x}{1 + \tan x} = \tan\left(\frac{\pi}{4} - x\right)$

$\Rightarrow \frac{dy}{dx} = -\sec^2\left(\frac{\pi}{4} - x\right)$

6. (c)

We have,

$A = \int_0^{\pi/4} \sin x \, dx = [-\cos x]_0^{\pi/4} = \frac{\sqrt{2} - 1}{\sqrt{2}} = 1 - \frac{1}{\sqrt{2}}$ ... (i)

Let A_1 be the required area. Then,

$A_1 = \int_0^{\pi/4} \cos x \, dx = [\sin x]_0^{\pi/4} = \frac{1}{\sqrt{2}}$ ... (ii)

From (i) and (ii), we have

Required area $A_1 = 1 - A$

7. (d)

Let the point in xy-plane be $P(x_1, y_1, 0)$. Let the given points are $A(2,0,3)B(0,3,2)$

And $C(0,0,1)$

According to the given condition,

$AP^2 = BP^2 = CP^2$

$\therefore (x_1-2)^2+y_1^2+9=x_1^2+(y_1-3)^2+4$
$=x_1^2+y_1^2+1$
From Ist and IInd terms,
$x_1^2+4-4x_1+y_1^2+9=x_1^2+y_1^2-6y_1+9+4$
$\Rightarrow 4x_1-6y_1=0$ …..(i)
From IInd and IIIrd terms,
$x_1^2+y_1^2+9-6y_1+4=x_1^2+y_1^2+1$
$\Rightarrow 6y_1=12 \Rightarrow y_1=2$
On putting the value of y_1 in Eq.(i), we get $x_1=3$
Hence, required point is (3, 2, 0).

8. (d)

Given, $\sqrt{(3x+1)^2}<(2-x)$
$\Rightarrow (3x+1)<2-x$
$\Rightarrow 3x+1<2-x \Rightarrow x<\frac{1}{4}$

9. (b)

We have,
$\int_1^e (\log x)^3 dx=[x(\log x)^3]_1^e-3\int_1^e (\log x)^2 dx$
$\Rightarrow \int_1^e (\log x)^3 dx=e-3\left[\{x(\log x)^2\}_1^e-2\int_1^e (\log x)dx\right]$
$\Rightarrow \int_1^e (\log x)^3 dx=e-3[e-2[x\log x-x]_1^e]=6-2e$

10. (b)

Total number of ways placing 3 letters in three envelopes
$=3!=3\times2\times1=6$
Out of these ways only one way is correct
$\therefore$ The required probability $=\frac{1}{6}$

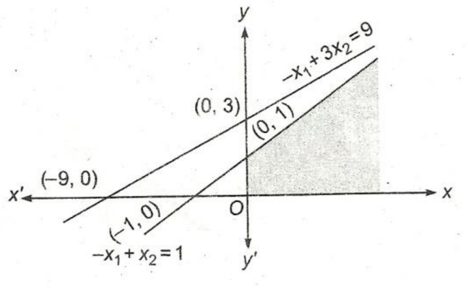
11. (c)

We have,
$C_0^2+3\cdot C_1^2+5\cdot C_2^2+\cdots+(2n+1)C_n^2$
$=\{C_0^2+C_1^2+C_2^2+\cdots+C_n^2\}$
$+\{2\,C_1^2+4\cdot C_2^2+6\cdot C_3^2+\cdots+2\,n\,C_n^2\}$ ... (i)
We have,
$(1+x)^{2n}=(1+x)^n(1+x)^n$
$\Rightarrow (1+x)^{2n}=(C_0+C_1x+C_2x^2+\cdots+C_nx^n)\times(C_0x^n+C_1x^{n-1}+\cdots+C_{n-1}\,x+C_n)$
On equating the coefficient of x^n on both sides, we get
${}^{2n}C_n=C_0^2+C_1^2+C_2^2+\cdots C_n^2$ ... (ii)
Also,
$n(1+x)^{n-1}(1+x)^n=(C_1+2\,C_2\,x+3\,C_3x^2+\cdots+n\,C_nx^{n-1})$

$\times (C_0 x^n + C_1 x^{n-1} + C_2 x^{n-2} + \cdots + C_n)$
On equating the coefficient of x^{n-1} on both sides, we get
$n \cdot {}^{2n-1}C_{n-1} = (C_1^2 + 2\,C_2^2 + 3\,C_3^2 + \cdots + n\,C_n^2)$
$\Rightarrow 2n \cdot {}^{2n-1}C_{n-1} = 2\,C_1^2 + 4\,C_2^2 + 6\,C_3^2 + \cdots + +2\,n\,C_n^2$ ... (iii)
From (i),(ii) and (iii), we obtain
$C_0^2 + 3 \cdot C_1^2 + 5\,C_2^2 + \cdots + (2n+1)C_n^2$
$= \frac{2n}{n}\,{}^{2n-1}C_{n-1} + 2n \cdot {}^{2n-1}C_{n-1} = 2(n+1)\,{}^{2n-1}C_{n-1}$

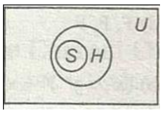
12. (b)

It is clear from the figure that feasible space (shaded portion)
is unbounded



13. (c)

The required venn diagram of given statement is



14. (c)

Equation of perpendicular diagonal to $7x - y + 8 = 0$ is $x + 7y = \lambda$, which passes through $(-4, 5)$
$\therefore\ \lambda = 31$
So, equation of another diagonal is
$x + 7y = 31$

15. (b)

Total number of points are $m + n + k$, the triangles formed by these points $= {}^{m+n+k}C_3$
Joining of three points on the same line gives no triangle, the number of such triangles is ${}^{m}C_3 + {}^{n}C_3 + {}^{k}C_3$
$\therefore$ Required number of triangles
$= {}^{m+n+k}C_3 - {}^{m}C_3 - {}^{n}C_3 - {}^{k}C_3$

16. (d)
17. (c)

$\cos^2 A(3 - 4\cos^2 A)^2 + \sin^2 A(3 - 4\sin^2 A)^2$
$= (3\cos A - 4\cos^3 A)^2 + (3\sin A - 4\sin^3 A)^2$
$= (-\cos 3A)^2 + (\sin 3A)^2 = 1$

18. (b)

Given, $f(x) = x^3 - 3x^2 + 2x$

$\Rightarrow f'(x) = 3x^2 - 6x + 2$

Now, f(a)=f(0)=0

And $f(b) = f\left(\frac{1}{2}\right)$

$= \frac{1}{2}\left(\frac{1}{2} - 1\right)\left(\frac{1}{2} - 2\right) = \frac{3}{8}$

By Lagrange's Mean Value Theorem

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

$$\Rightarrow \frac{\frac{3}{8} - 0}{\frac{1}{2} - 0} = 3c^2 - 6c + 2$$

$\Rightarrow 12c^2 - 24x + 5 = 0$

This is a quadratic equation in c.

$$c = \frac{24 \pm \sqrt{576 - 240}}{24}$$

$$= 1 \pm \frac{\sqrt{21}}{6}$$

But c lies between 0 to$\frac{1}{2}$

$\therefore$ we take, $c = 1 - \frac{\sqrt{21}}{6}$

19. (b)

We have,

$f(x) = |x - 1| + |x - 3|$

$$\Rightarrow f(x) = \begin{cases} -(x-1) - (x-3), & x < 1 \\ (x-1) - (x-3), & 1 \le x < 3 \\ (x-1) + (x-3), & x \ge 3 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} -2x + 4, & x < 1 \\ 2, & 1 \le x < 3 \\ 2x - 4, & x \ge 3 \end{cases}$$

Since, $f(x) = 2$for $1 \le x < 3$. Therefore $f'(x) = 0$ for all $x \in (1, 3)$

Hence, $f'(x) = 0$ at $x = 2$

20. (b)

$$\frac{[(\cos 20° + i \sin 20°)(\cos 75° + i \sin 75°)(\cos 10° + i \sin 10°)]}{\sin 15° - i \cos 15°}$$

$$= \frac{e^{i20°} e^{i75°} . e^{i10°}}{-i(\cos 15° + i \sin 15°)}$$

$$= -\frac{e^{i105°}}{ie^{i15°}}$$

$$= -\frac{e^{i90°}}{i} = -1$$

21. (3)

$$L = \lim_{x \to 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$$

$= -\lim_{x\to 0}\frac{(1-\cos x)(1+\cos x)(\cos x - e^x)}{(1+\cos x)x^n}$

$= \lim_{x\to 0}\frac{\left(\frac{\sin x}{x}\right)^2\left(\frac{1-\cos x}{x}+\frac{e^x-1}{x}\right)}{x^{n-3}}\frac{1}{1+\cos x}$

If L is finite non-zero, then $n = 3$ (as for $n = 1, 2, L = 0$ and for $n = 4, L = \infty$)

22. (8)

Let $D = \begin{vmatrix}(\beta+\gamma-\alpha-\delta)^4 & (\beta+\gamma-\alpha-\delta)^2 & 1\\ (\gamma+\alpha-\beta-\delta)^4 & (\gamma+\alpha-\beta-\delta)^2 & 1\\ (\alpha+\beta-\gamma-\delta)^4 & (\alpha+\beta-\gamma-\delta)^2 & 1\end{vmatrix}$

Applying $R_1 \to R_1 - R_3, R_2 \to R_2 - R_3$

$= \begin{vmatrix}(\beta+\gamma-\alpha-\delta)^4-(\alpha+\beta-\gamma-\delta)^4 & (\beta+\gamma-\alpha-\delta)^2-(\alpha+\beta-\gamma-\delta)^2 & 0\\ (\gamma+\delta-\beta-\delta)^4-(\alpha+\beta-\gamma-\delta)^4 & (\gamma+\alpha-\beta-\delta)^2-(\alpha+\beta-\gamma-\delta)^2 & 0\\ (\alpha+\beta-\gamma-\delta)^4 & (\alpha+\beta-\gamma-\delta)^2 & 1\end{vmatrix}$

$= 4(\beta-\delta)(\gamma-\alpha).4(\alpha-\delta)(\gamma-\beta)$

$\times\begin{vmatrix}(\beta+\gamma-\alpha-\delta)^2+(\alpha+\beta-\gamma-\delta)^2 & 1 & 0\\ (\gamma+\alpha-\beta-\delta)^2+(\alpha+\beta-\gamma-\delta)^2 & 1 & 0\\ (\alpha+\beta-\gamma-\delta)^4 & (\alpha+\beta-\gamma-\delta)^2 & 1\end{vmatrix}$

Apply $R_1 \to R_1 - R_2$

$= 16(\beta-\delta)(\gamma-\alpha)(\alpha-\delta).4(\gamma-\delta)(\beta-\alpha)$

$\begin{vmatrix}1 & 0 & 0\\ (\gamma+\alpha-\beta-\delta)^2+(\alpha+\beta-\gamma-\delta)^2 & 1 & 0\\ (\alpha+\beta-\gamma-\delta)^4 & (\alpha+\beta-\gamma-\delta)^2 & 1\end{vmatrix}$

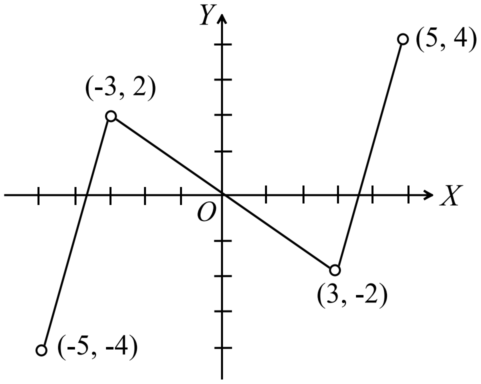
$= -64(\alpha-\beta)(\alpha-\gamma)(\alpha-\delta)(\beta-\gamma)(\beta-\delta)(\gamma-\delta)$

23. (3)

$f(x) + f(-x) = 0$

$\Rightarrow f(x)$is an odd function.

Since point $(-3, 2)$ and $(5,4)$ lie on the curve, therefore $(3, -2)$ and $(-5, -4)$ will also lie on the curve. For minimum number of roots, graph of continuous function $f(x)$ is as follows.



From the above graph of $f(x)$, it is clear that equation $f(x)$=0 has at least three real roots.

24. (2)

L.H.S $= \vec{d} - \vec{a} + \vec{d} - \vec{b} + \vec{h} - \vec{c} + 3(\vec{g} - \vec{h})$

$= 2\vec{d} - (\vec{a} + \vec{b} + \vec{c}) + 3\dfrac{(\vec{a} + \vec{b} + \vec{c})}{3} - 2\vec{h}$

$= 2\vec{d} - 2\vec{h} = 2(\vec{d} - \vec{h}) = 2\vec{H}D$

$\Rightarrow \lambda = 2$

25. (9)

Given $S = \sum_{n=1}^{9999} \frac{1}{(\sqrt{n}+\sqrt{n+1})(\sqrt[4]{n}+\sqrt[4]{n+1})}$

$$= \sum_{n=1}^{9999} \frac{1}{(\sqrt{n} + \sqrt{n+1})(\sqrt[4]{n} + \sqrt[4]{n+1})} \left(\frac{\sqrt[4]{n} - \sqrt[4]{n+1}}{\sqrt[4]{n} - \sqrt[4]{n+1}}\right)$$

$$= \sum_{n=1}^{9999} \left((n+1)^{1/4} - n^{1/4}\right)$$

$$= \left(\left(2^{\frac{1}{4}} - 1\right) + \left(3^{\frac{1}{4}} - 2^{\frac{1}{4}}\right) + \left(4^{\frac{1}{4}} - 3^{\frac{1}{4}}\right) + \cdots + \left((9999+1)^{\frac{1}{4}} - (9999)^{\frac{1}{4}}\right)\right)$$

$= (10^4)^{\frac{1}{4}} - 1 = 9$

26. Ans. 0

Sol. $[-2\pi] = -7$ $\quad \therefore$ L.H.S. is positive

R.H.S. is negative $\quad \therefore$ no solution

27. Ans. 1999

Sol. $(1-x)^n = {}^nC_0 - {}^nC_1x + {}^nC_2x^2 - \ldots\ldots {}^nC_nx^n$

multiplying both sides by x

$x(1-x)^n = {}^nC_0x - {}^nC_1x^2 + {}^nC_2x^3 - \ldots\ldots {}^nC_n x^{n+1}$

Integrating both sides

$$\int_0^1 x(1-x)^n = \int_0^1 {}^nC_0x - {}^nC_1x^2 + {}^nC_2x^3 + \ldots\ldots + {}^nC_nx^{n+1}$$

$$\frac{1}{(n+1)(n+2)} = \frac{{}^nC_0}{2} - \frac{{}^nC_1}{3} + \frac{{}^nC_2}{4} \ldots.$$

$\Rightarrow \dfrac{1}{(n+1)(n+2)} = \dfrac{1}{2000 \times 2001}$ i.e. n = 1999

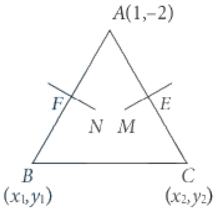
28. Ans. 0001

Sol. $\dfrac{1}{81^n}\left[1 - 10\ {}^{2n}C_1 + 10^2 .\ {}^{2n}C_2 - 10^3\ {}^{2n}C_3 \ldots.. + 10^{2n}\right]$

$= \dfrac{1}{81^n}[1-10]^{2n} = \dfrac{81^n}{81^n} = 1$

29. 63

Let the equation of perpendicular bisector FN of AB is $x - y + 5 = 0$ ... ... (i)



The middle point F of AB is $\left(\frac{x_1+1}{2}, \frac{y_1-2}{2}\right)$, lies on line (i). Therefore $x_1 - y_1 = -13 \ldots (ii)$

Also AB is perpendicular to FN. So the product of their slopes is -1.

i.e. $\frac{y_1+2}{x_1-1} \times 1 = -1$ or $x_1 + y_1 = -1 \ldots (iii)$

On solving (ii) and (iii), we get $B(-7,6)$.

Similarly, $C\left(\frac{11}{5}, \frac{2}{5}\right)$

Hence the equation of BC is $14x + 23y - 40 = 0$.

$a = 23, b = 40$

$a + b = 63$

30. Ans. 4

Sol. $\therefore 5^{-P} = 5^{-\log_5 \log_5(3)} \Rightarrow 5^{-P} = \log_3 5 \Rightarrow 3^{C+\log_3 5} = 405$

$(3^C)(5) = 405 \Rightarrow 3^C = 81 \Rightarrow C = 4$