1. By the conservation of mechanical energy,

$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$4 \times (2)^2 = 100 \times x^2$

$\frac{4 \times 4}{100} = x^2$

$\Rightarrow \frac{16}{100} = x^2$

$\Rightarrow x = 0.4$ m

2. Given that two particles having same specific charge are accelerated through same potential difference and are released into a magnetic field, which is perpendicular to their velocities.

After accelerating a particle through V, particle gains kinetic energy equal to qV.

$\frac{1}{2}mv^2 = qV$

$\Rightarrow v = \sqrt{\frac{2qV}{m}}$

$\Rightarrow v = \sqrt{2\rho V}$, where ρ is a specific charge.

Radius of circular path followed by particle in magnetic field region, $r = \frac{mv}{qB}$

$\Rightarrow r = \frac{\sqrt{2\rho V}}{\rho B}$

$\Rightarrow r = \sqrt{\frac{2V}{\rho}}\frac{1}{B}$

As ρ, B and V for both the particles are same, ratio of radius will be 1 : 1.

3. $v = \sqrt{\frac{N}{\mu}}$

The tension N in the string varies as :

$N = \frac{Mgx}{L}$ where x is length from the ground.

$dt = \frac{dx}{v_x}$ and $v_x = \sqrt{\frac{Mgx}{L \times \frac{M}{L}}} = \sqrt{gx}$

$\int_0^T dt = \int_0^L \frac{dx}{\sqrt{gx}}$

$T = \frac{2\sqrt{L}}{\sqrt{g}}$ ... (i)

If time to cover half length is T_2.

$T_2 = \frac{2\sqrt{L}}{\sqrt{2g}}$

$\frac{T}{\sqrt{2}} = T_2$

4. Given that water droplets are coming from an open tap at a particular rate.

The spacing between a droplet observed at 4^{th} second after its fall to the next droplet is 34.3 m.

Consider droplets are coming from an open tap at a rate of N drop/sec. i.e time gap two consecutive droplets is $\frac{1}{N}$ sec.

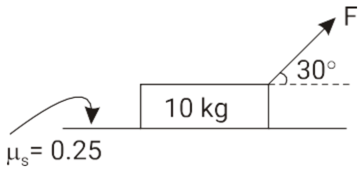
At t = 4 sec after the fall of a droplet, gap with next droplet is, $\Delta H = \frac{1}{2}g(4)^2 - \frac{1}{2}g\left(4 - \frac{1}{N}\right)^2$

$\Rightarrow 34.3 = 9.8 \times \left(4 - \frac{1}{N}\right) - (4.9)\frac{1}{N^2}$

$\Rightarrow 5 = 2 \times \left(4 - \frac{1}{N}\right) - \frac{1}{N^2}$

$\Rightarrow \frac{1}{N^2} + \frac{2}{N} - 3 = 0$

$\Rightarrow N = 1$

5. 

$F\cos 30° = \mu N$

$N = 10g - F\sin 30° = 100 - \frac{F}{2}$

On solving (i) and (ii), we get,

$\frac{\sqrt{3}}{2}F = 0.25\left(100 - \frac{F}{2}\right)$

$\Rightarrow \left(\frac{\sqrt{3}}{2} + \frac{1}{8}\right)F = 25$

$F = \frac{25 \times 8}{(1 + 4\sqrt{3})} = \frac{200}{(4\sqrt{3} + 1)}$ N

= 25.22 N

6. An atom being a spherical cloud of positive charges with electrons embedded in it, is J. J. Thomson's model of the atom and not Rutherford's model.

7. $k = 6 \times 10^5$ N/m

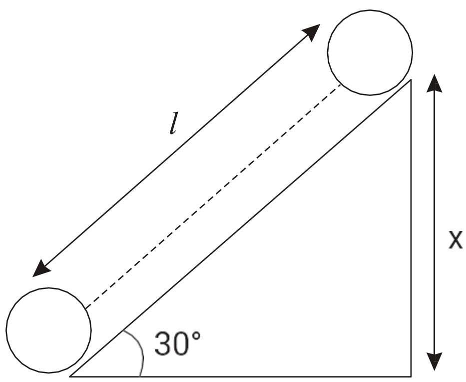
Amplitude = 4 cm = 4×10^{-2} m

$\frac{1}{2}kA^2 = \frac{1}{2} \times 6 \times 10^5 \times (4 \times 10^{-2})^2$

or E = 480 J

So, 480 J is the oscillation energy. The potential energy at the mean position is not zero in this case. It is equal to 600 - 480 = 120 J. 480 J keeps on oscillating between the kinetic energy and potential energy.

8. Paramagnetism is temperature dependent whereas diamagnetism is not.

9. 

By energy conservation

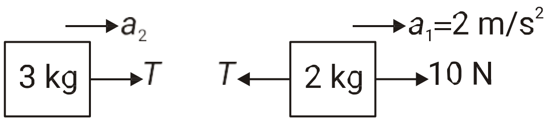
$\frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right) = mgx = mg\,l\sin\vartheta$

So $l = \frac{v^2(1+\frac{K^2}{R^2})}{2g\sin\vartheta}$

$l = \frac{(4)^2 \times (1+\frac{1}{2})}{2 \times 10 \times \frac{1}{2}}$ = 2.4 meter

10. The velocity of the sphere colliding with the xz surface is, $\vec{v} = a\hat{i} - b\hat{j}$.
The coefficient of restitution between the sphere and the flat surface = e.
The component of the velocity along the common normal before the collision = $-b\hat{j}$.
The wall is smooth. It can exert a force on the sphere only along the common normal, not along the surface.
So, the normal component, i.e., the y-component reverses and becomes e times and the tangential component, i.e., the x-component remains the same.
[coefficient of restitution, e = $\frac{\text{velocity of separation along the common normal}}{\text{velocity of approach along the common normal}}$]
⇒ The component of velocity along the common normal after the collision = $e b\hat{j}$.
⇒ The component of the velocity along the tangential direction = $a\hat{i}$.
∴ The final velocity after the collision = $a\hat{i} + eb\hat{j}$.

11. As blocks are connected by spring, which is extensible, blocks may possess diffferent accelerations.
F.B.D of blocks :



For 2 kg mass ,10 – T = 2 × 2 [Newton's 2nd law, considering forward directions as positive]

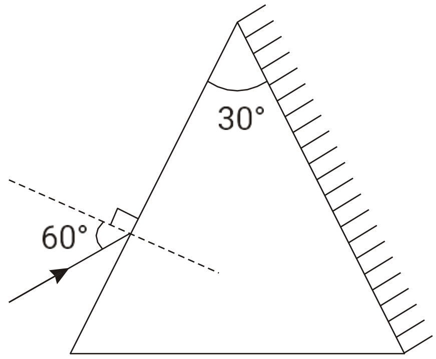
⇒ T = 6 N

For 3 kg mass , $T = 3 \times a$

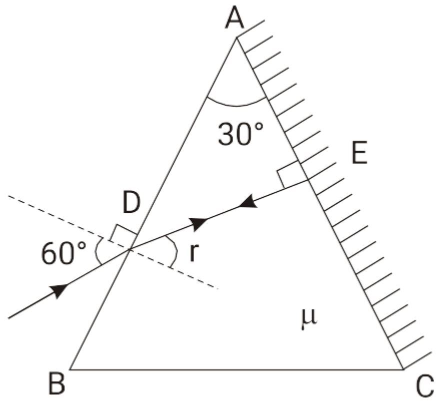
⇒ $6 = 3a$

⇒ $a = 2$ m/s^2

12. Given that an isosceles prism of angle $A = 30^\circ$ has one of its surfaces silvered. Light rays falling at an angle of incidence 60° on the other surface retrace their path after reflection from the silvered surface. We have to find the refractive index of the prism material.



For light to retrace its path, it must be incident normally on the silvered surface as shown below.



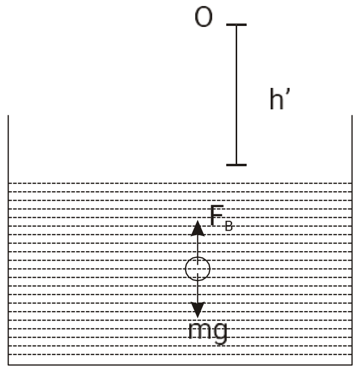
Now, $\angle ADE = 90^\circ - r$.
From ΔADE, $30^\circ + 90^\circ - r + 90^\circ = 180^\circ$
$\Rightarrow r = 30^\circ$
From Snell's Laws of Refraction at D, we get,
$(1)\sin 60^\circ = \mu \sin 30^\circ$
$\Rightarrow \mu = \frac{(\frac{\sqrt{3}}{2})}{(\frac{1}{2})} = \sqrt{3}$

13. Given, mass of body A = m, mass of body B = $\frac{m}{2}$.
Velocity of body A before collision = $\vec{v}$
Velocity of body B before collision = $\frac{\vec{v}}{2}$
As collision is headon, completely inelastic, both the bodies travel with common velocity after the collision. Consider, v_f is the final velocity of the combined mass.
By Conserving momentum we can write,
$\frac{m}{2}\frac{v}{2} + mv = (m + \frac{m}{2})v_f$
$\Rightarrow v_f = \frac{5mv}{4 \times \frac{3m}{2}} = \frac{5v}{6}$
As $v_f < v_{orb}(= v)$, the combined mass will go on to an elliptical path.

14. Given that when the angle of incidence from air on a material is 60°, the reflected light is completely polarized. We have to find the velocity of the refracted ray inside the material (in ms^{-1}).
The angle of incidence (ϑ_i) in this case is the Brewster's angle (ϑ_B).
Now, $\tan\vartheta_B = \mu$.
$\Rightarrow \mu = \tan 60^\circ = \sqrt{3}$
Also, $\mu = \frac{c}{v}$, where c = the velocity of light in vaccum and v = the velocity of light in the medium.
So, $\sqrt{3} = \frac{c}{v}$
$\Rightarrow v = \frac{c}{\sqrt{3}} = \frac{3 \times 10^8}{\sqrt{3}}$
$\Rightarrow v = \sqrt{3} \times 10^8$ m/s

15. Given that a ball of mass m and density ρ is immersed in a liquid of density 3ρ at a depth h and released.

While the body is inside the liquid, ball experience both gravitational force and buoyancy force due to surrounding liquid. In the given case, density of liquid is greater than density of body. So buoyancy force is greater than weight of the object.



Net upward force = $F_B - mg$

$\Rightarrow \Sigma \vec{F} = v(3\rho)g - v\rho g$

$\Rightarrow \Sigma \vec{F} = 2v\rho g$

Acceleration of ball inside the liquid, $a_{c_e} = \frac{2v\rho g}{m} = 2g$.

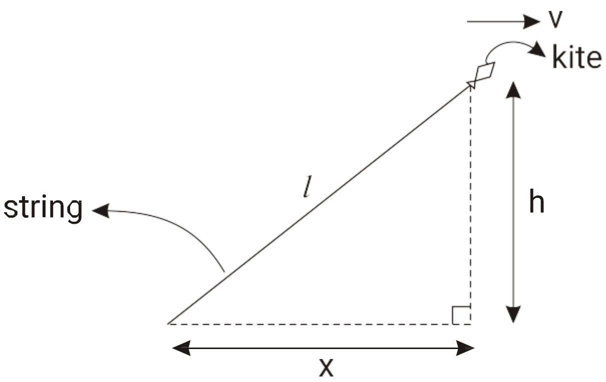
Let the velocity of ball is v m/s when it is leaving the liquid.
Applying 3rd equation of kinematics, $v^2 = 0 + 2(2g)h$

Once ball leaves the liquid, it experiences only gravitational force. Motion of the ball will be similar to the body projected vertically upwards. During this journey, ball have downward acceleration (g). Let ball comes to rest after travelling a distance of h' in air.

Applying 3rd equation of kinematics, $0 = 2(2g)h - 2gh'$

$\Rightarrow h^1 = 2h$.

16. Given that a kite is 120 m high and 130 m of string is out. If the kite is moving away horizontally at the rate of 52 m/s, we have to find the rate at which the string is being pulled.



Let at any moment, the length of the string be l.
Then by Pythagoras' Theorem, we get, $l^2 = x^2 + h^2$.
Differentiating both sides with respect to time, we get, $2l\frac{dl}{dt} = 2x\frac{dx}{dt} + 0$. (The height h of the kite above the ground is constant)

$\Rightarrow \frac{dl}{dt} = \left(\frac{x}{l}\right)\left(\frac{dx}{dt}\right)$

Now, $\frac{dx}{dt}$ = v = the rate at which the kite is being pulled away horizontally.

Also, $x = \sqrt{l^2 - h^2} = \sqrt{130^2 - 120^2} = 50$ m.

So, we get, $\frac{dl}{dt} = \left(\frac{x}{l}\right)v$.

$\Rightarrow \frac{dl}{dt} = \left(\frac{50}{130}\right)(52)$

$\Rightarrow \frac{dl}{dt} = 20$ m/s

So, the rate at which the string in pulled out is 20 m/s.

17. Given that in an AC circuit, the instantaneous voltage $e(t)$ and current $I(t)$ are given by
$e(t) = 5[\cos \omega t + \sqrt{3}\sin \omega t]$ volt, $i(t) = 5\left[\sin\left(\omega t + \frac{\pi}{4}\right)\right]$ amp respectively.
Voltage as a single sine function can be written as,

$e(t) = 10\left[\frac{\sqrt{3}}{2}\sin \omega t + \frac{1}{2}\cos \omega t\right]$

$\Rightarrow e(t) = 10\left[\sin\left(\omega t + \frac{\pi}{6}\right)\right]$

Phase difference between the current and voltage is,
$\Delta\phi = \frac{\pi}{4} - \frac{\pi}{6}$

$\Rightarrow \Delta\phi = \frac{3\pi - 2\pi}{12}$

$\Rightarrow \Delta\phi = \frac{\pi}{12}$, with current leading voltage.

18. Elongation produced in the string can be written as $\Delta l = \frac{FL}{AY}$

Internal restoring force developed in steel = $7mg$ and in copper = $5mg$.

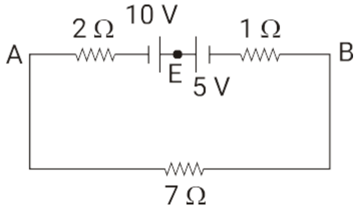
$\Rightarrow \Delta l_{steel} = \frac{(5m + 2m)gL_{steel}}{A_{steel} \cdot Y_{steel}}$

$\Rightarrow \Delta l_{cu} = \frac{(5m)g(L_{Cu})}{A_{cu} \cdot Y_{cu}}$

$\Rightarrow \frac{\Delta l_{steel}}{\Delta l_{cu}} = \frac{7}{5}\frac{L_{steel}}{L_{Cu}}\frac{A_{Cu}}{A_{steel}}\frac{Y_{Cu}}{Y_{steel}}$

$\therefore \frac{\Delta l_{steel}}{\Delta l_{cu}} = \frac{7}{5}\frac{q}{sp^2}$ [Ratio of diameters is p. So, ratio of areas will be equal to p^2]

19. $i = \frac{10 - 5}{10} = \frac{5}{10}$ A



i = 0.5 A from A to B through E.

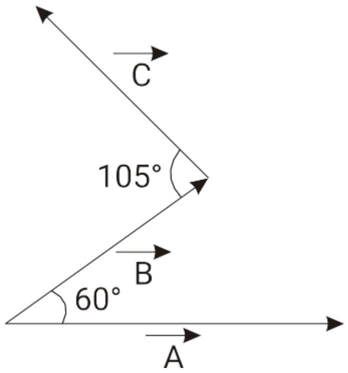
20. Given that the electric potential V at a point P(x, y, z) in space is given by V = $4x^2$ Volt. We have to find the electric field at a point (1m, 0, 2m) in V/m.

The components of the electric field can be written as,
$E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y} = 0$, $E_z = -\frac{\partial V}{\partial z} = 0$.
The y and z components are zero because the potential function is a sole function of x and does not contain y or z in it. (Look up partial differentiation.)

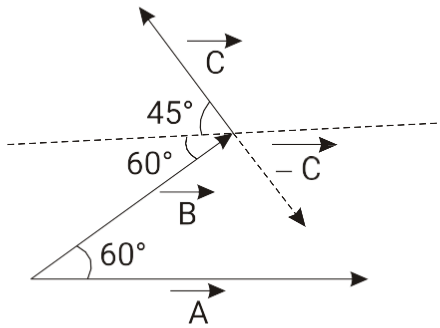
So, $E_x = -\frac{d}{dx}(4x^2)$.

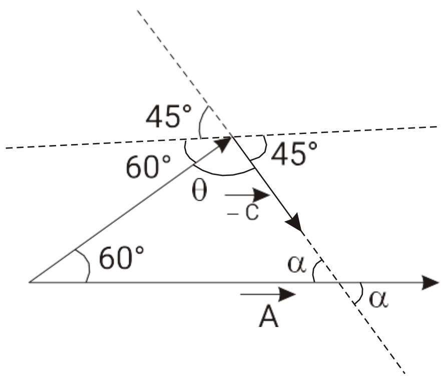
$\Rightarrow E_x = -8x = -8(1) = -8$ V/m

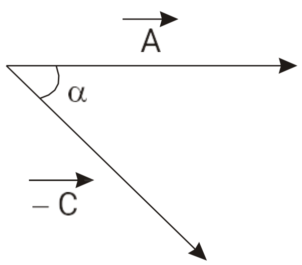
21. In the given figure, if the angle between the vectors A and −C is $5\alpha^\circ$, then we have to find the value of α.

2 

The situation is shown below.







The angle between two vectors is obtained by matching their tails.
So, the required angle here is α.
Now, $\vartheta = 180^\circ - 105^\circ = 75^\circ$.
Also, $\alpha + \vartheta + 60^\circ = 180^\circ$
$\Rightarrow \alpha = 120^\circ - 75^\circ = 45^\circ$

$\Rightarrow 5\alpha = 45$

$\Rightarrow \alpha = 9$

22. Centrifugal force acts on the particle in the radially outward direction.

$\frac{vdv}{dx} = \omega^2 x$

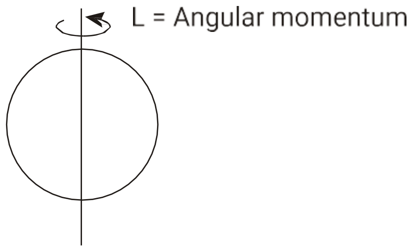
$\int_0^v vdv = \int_1^3 \omega^2 x dx$

$\frac{v^2}{2} = \omega^2\left[\frac{x^2}{2}\right]_1^3$

$\frac{v^2}{2} = \frac{\omega^2}{2}[3^2 - 1^2]$

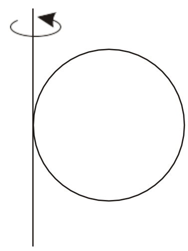
$v = 2\sqrt{2}\omega$

v = 2 m/s

23. 

m = 500 g = 0.5 kg
R = 5 cm
ω = 10 rad/sec

Moment of inertia about tangent = I_T



$I_T = x \times 10^{-2} L$

$\frac{7}{5} mR^2 = x \times 10^{-2} \times \frac{2}{5} mR^2\omega$

$\frac{7}{2\omega} = x \times 10^{-2} = \frac{7}{2 \times 10}$

$x = 35$

24. Apparent depth = $\frac{d_1}{\mu_1} + \frac{d_2}{\mu_2}$

$= \frac{6}{\left(\frac{3}{2}\right)} + \frac{6}{\left(\frac{8}{5}\right)} = \frac{31}{4}$ cm

25. $W = \int_0^4 (2 + 3x)dx$

$= \left[2x + \frac{3x^2}{2}\right]_0^4$

$= 8 + 3 \times 8$
$= 32$ J

26. mass of SO_2 taken = 64 g

Gram molecular weight of SO_2 = 64 g/mol

⇒ moles of SO_2 taken = $\frac{mass}{M.W} = \frac{64}{64} = 1$

Number of molecules of SO_2 present in 1 mole of SO_2 = 6.02×10^{23}

SO_2 is a tri-atomic molecule

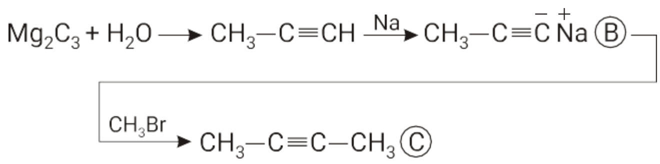
Each molecule of SO_2 contains 1 atom of S and 2 atoms of O i.e. a total of 3 atoms

Hence, number of atoms present in 64 g SO_2 = 3 × number of molecules of SO_2

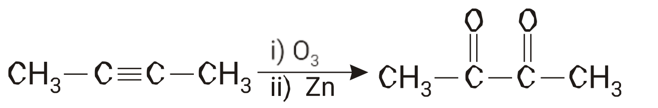
⇒ number of atoms present in 64 g SO_2 = $3 \times 6.02 \times 10^{23}$???????

27. The steps in the given sequence of reactions are:
Step 1: Magnesium carbide upon hydrolysis gives propyne.
Step 2: The terminal H of propyne makes salt with metal, Na.
Step 3: The salt of propyne attacks CH_3Br and nucleophilic substitution reaction takes place.

The product formed in each step has been shown below:



In the compound (C) the middle two carbon atoms are sp hybridized and hence all the four carbons atoms are linear.
Tollen test is given by aldehyde and terminal alkyne also gives ppt. (of Ag salt) with Tollen's reagent. But compound (C) is non-terminal alkyne so it does not give positive Tollen test.
Compound (C) on ozonolysis gives 1,2 diketo compound as shown:



1, 2 diketo compound

⇒ The statement given in option (B) is not true.
⇒ Option (B) is *CORRECT*.

28. The balanced equation of $Ca(OH)_2$ and sodium sulphate is given below:

$Na_2SO_4 + Ca(OH)_2 \rightarrow CaSO_4 + 2NaOH$

m.moles of $Ca(OH)_2$ taken = 100

m.mol of $Na_2SO_4 = \frac{2 \times 1000}{143} \simeq 13.98$

⇒ Na_2SO_4 is a limiting reagent.
⇒ m.mol of $CaSO_4$ formed = 13.98

⇒ Mass of $CaSO_4$ formed = $13.98 \times 10^{-3} \times 136$ = *1.90 g*

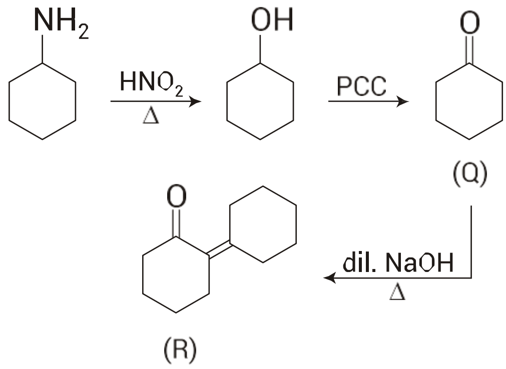
⇒ m.mol of NaOH = 13.98 ×2 ≃ 28 mmol

The equilibrium of $Ca(OH)_2$ can be represented as,
$Ca(OH)_2 \rightleftharpoons \underset{S}{Ca^{2+}} + \underset{2S+0.28}{2OH^-}$

But NaOH is a strong base, so the concentration of OH^- will be decided by the NaOH.
⇒ m.moles of OH^- in 100 ml = 28 mmol
⇒ moles of OH^- in 100 ml = 0.028
⇒ $[OH^-] = \frac{0.028}{0.1} = 0.28$ mol L^{-1}

⇒ Option (D) is *CORRECT.*

29.



30. Zn dust do E_2 elimination of X_2, hence a vicinal dihalide will give alkene.

$CH_2Br - CH_2Br \xrightarrow{\text{Zn dust}} \underset{\text{ethylene}}{CH_2 = CH_2}$

$CHBr_2 - CHBr_2 \xrightarrow{\text{Zn dust}} \underset{\text{acetylene}}{CH \equiv CH}$

$CHBr = CHBr \xrightarrow{\text{Zn dust}} \underset{\text{acetylene}}{CH \equiv CH}$

hence answer is: $CH_2Br—CH_2Br$

31. This test is applied to those salts that leave white residue in the charcoal cavity test.
This test is based on the fact that metallic carbonates when heated in a charcoal cavity decomposes to give corresponding oxides.
The test is based on the fact that cobalt nitrate decomposes on heating to give cobalt oxide, CoO.
This combines with the metallic-oxides present as a white residue in the charcoal cavity-forming colored compounds.

$2Co(NO_3)_2 \xrightarrow{\Delta} 2CoO + 4NO_2 + O_2$

$Al_2O_3 + CoO \rightarrow Al_2O_3.CoO_{(\text{Blue color})}$

⇒ $Al_2O_3.CoO$ formed in the test is blue in color.
⇒ Option (D) is *CORRECT.*

32. A molecule exista only if the bond order is positive. If bond order is zero or negative, the molecule does not

exist.

33. Statement IV is incorrect.
Its correct form is as follows:
The valence of representative elements is usually equal to the number of electrons in the outermost orbitals and/or equal to eight minus the number of outermost electrons.
Rest other statements are correct.

34. Given reaction:

	$2\,AB_2$ $\rightleftharpoons$	$2\,AB$ +	B_2
t = 0	1	0	0
Equilibrium	$1-x$	x	$\frac{x}{2}$
Mole fraction	$\frac{1-x}{1+\frac{x}{2}}$	$\frac{x}{1+\frac{x}{2}}$	$\frac{\frac{x}{2}}{1+\frac{x}{2}}$
Partial Pressure	$\left(\frac{1-x}{1+\frac{x}{2}}\right)P$	$\left(\frac{x}{1+\frac{x}{2}}\right)P$	$\left(\frac{\frac{x}{2}}{1+\frac{x}{2}}\right)P$

Assuming the initial moles of AB_2 = 1
Moles of AB_2 dissociated at equilibrium = x
Total number of moles of gases at equilibrium = $(1 - x) + x + \frac{x}{2} = 1 + \frac{x}{2}$

We know, Degree of dissociation = $\frac{\text{Number of moles dissociated}}{\text{Number of moles taken}}$
⇒ x = Degree of dissociation
The expression for K_p is given by,

$$K_p = \frac{P_{B_2} \times P^2_{AB}}{P^2_{AB_2}}$$

$$\Rightarrow K_p = \frac{\left(\frac{\frac{x}{2}}{1+\frac{x}{2}}\right)P \times \frac{x^2}{(1+\frac{x}{2})^2}P^2}{\frac{(1-x)^2}{(1+\frac{x}{2})^2}}$$

$$\Rightarrow K_p = \frac{Px^3}{2(1+\frac{x}{2})(1-x)^2}$$

Given, $x \ll 1$

$$\Rightarrow K_p = \frac{Px^3}{2 \times 1 \times 1} = \frac{Px^3}{2}$$

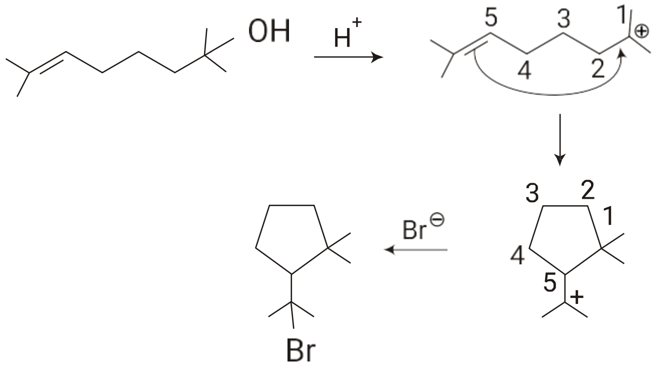
⇒ Option (A) is *CORRECT.*

35. The O.S. of Chromium in chromates, CrO_4^{2-}, is +6 (the highest allowable O.S. of Cr). HNO_3 is an oxidizing acid, thus it cannot reduce chromates to +3 O.S.
However, HNO_3 being an acid makes the solution acidic and we know that in acidic solution chromates change to dichromates and in basic solutions, dichromates change to chromates.
⇒ Excess dilute HNO_3 will convert CrO_4^{2-} to $Cr_2O_7^{2-}$

36. −Cl is deactivating group due to its −I effect but it direct electrophile to ortho and para position due to +M effect

37. $2Na_2S_2O_3 \longrightarrow Na_2S_4O_6 + 2NaI$
$2S_2^{4-} + 2e^- \longrightarrow S_4^{10-}$
2 electrons are gained by 2 moles of $Na_2S_2O_3$
⇒ number of electrons gained per mole of $Na_2S_2O_3$ = 1
⇒ n-factor of $Na_2S_2O_3$ = 1
We know that: Equivalent weight = $\frac{M.W.}{\text{n-factor}}$
Equivalent wt. of $Na_2S_2O_3$ = $\frac{M}{1}$ = M

38.



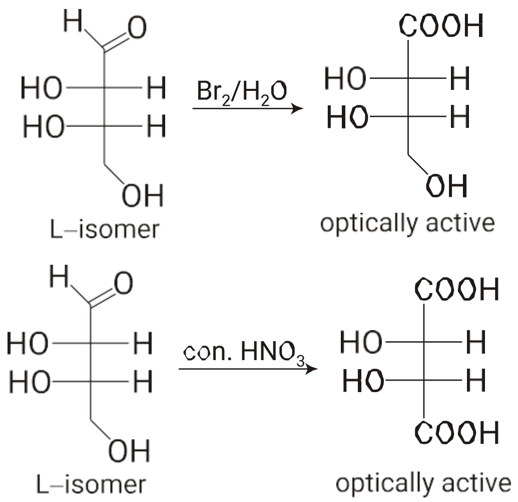
39. $Cl-Cl(g) \rightarrow 2Cl(g)$; ΔH = 242 KJ mol
$= \frac{242 \times 10^3}{6.02 \times 10^{23}}$ J molecule^{-1}
$E = \frac{hc}{\lambda}$
$\frac{242 \times 10^{-23} \times 10^3}{6.02} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{\lambda}$
$\lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{242 \times 10^{-23} \times 10^3} = \frac{6.6 \times 3 \times 6.02}{242} \times 10^{6} = 0.494 \times 10^{-6} = 494 \times 10^{-9}$ m = 494nm

40. (1) and (2) give aldol condensation, as it is given by aldehydes and ketones containing α-hydrogen atoms.
(3) gives Cannizzaro's reaction, as it is given by those aldehydes which do not contain α-hydrogen atom. (1) and (4) give haloform reaction, as it is given by those compounds which contain or make available, during the reaction, the CH_3CO group.

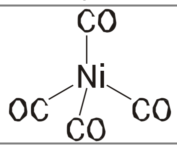
41. Lower the activation energy, faster is the reaction.

42. Dissolution of sugar in water results in increase in entropy, because disorderness of solid increases in solution.
$C_{12}H_{22}O_{11} + H_2O \rightarrow C_6H_{12}O_{6(glucose)} + C_6H_{12}O_6$ (fructose)
When egg is boiled, bonds are broken and egg attains a more disordered state
⇒ ΔS = +ve
Also, egg is boiled ⇒ spontaneous process.
So, ΔG < 0 at 100°C
∵ ΔH is +ve, ΔS must be +ve for ΔG to be −ve

43.



44.

$[NiCl_4]^{2-}$	$[Ni(CO)_4]$
sp^3	sp^3
Paramagnetic (2 unpaired electron)	
$Ni^{2+} \rightarrow [Ar]\ 3d^8, 4s^0, 4p^0$	$Ni(O) \rightarrow [Ar]\ 3d^8, 4s^2, 4p^0$
Cl^- (W.F.L.) (No pairing)	CO is S.F.L. $[Ar]\ 3d^{10}\ 4s^0, 4p^0$ sp^3 (Tetrahedral)

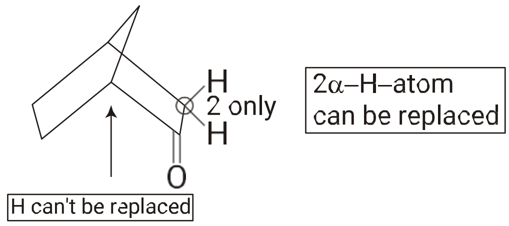
45.
$$A : M - ne \longrightarrow \underset{(0.02M)}{M^{n+}} \qquad E^o = +0.76$$
$$C : \underset{(1m)}{nH^+} + ne \longrightarrow \frac{n}{2}H_2(g) \qquad E^o = 0$$

$$M + nH^+ \longrightarrow M^{n+} + \frac{n}{2}H_2(g) \quad E^o = +0.76$$
$$E_{cell} = 0.81 = +0.76 - \frac{0.0591}{n}108\frac{[M^{n+}]}{[H^+]^n}$$
$$E_{cell} = 0.81 = +0.76 - \frac{0.0591}{n}\log\frac{[M^{n+}]}{[H^+]^n}$$
$$\frac{0.0591}{n} \times 1.7 = 0.81 - 0.76$$
$$n = \frac{0.0591 \times 1.7}{0.05} = 2.$$

46. The molar atomic weight of sodium, Na = 23 g mol^{-1}
⇒ Number of moles of Na atoms in 8 g of sodium sample = $\frac{8}{23}$
We know, 1 mole of any element has $N_a = 6.02 \times 10^{23}$ number of atoms.
⇒ Number of atoms of Na present in $\frac{8}{23}$ moles of Na =
$= \frac{8}{23} \times 6.02 \times 10^{23} = 2.09 \times 10^{23}$

⇒ Z = 2 (after rounding off to the nearest integer)
⇒ Answer = 2

47.

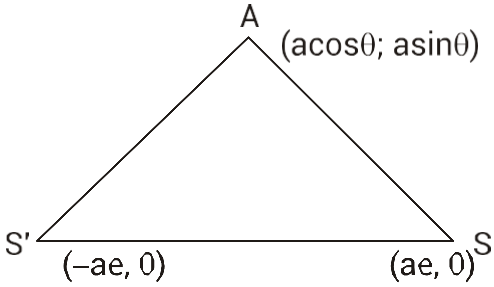


48. Let molarity of $KMnO_4 = M_1$
The molarity of $FeSO_4$ solution, $M_2 = 0.1$ M
Volume of $KMnO_4$ solution, V_1 = Volume of $FeSO_4$ solution, $V_2 = 10$ ml
The concerned redox reaction is:
$KMnO_4 + FeSO4 + H^+ \rightarrow Fe_2(SO_4)_3 + Mn^{2+}$
In acidic medium, $KMnO_4$ changes to Mn^{2+}
The oxidation number of Mn in $KMnO_4$ = +7
The oxidation number of Mn in Mn^{2+} = 2
The n–factor of $KMnO_4$, $n_1 = 1 \times (7 - 2) = 5$
In acidic medium, $FeSO_4$ changes to $Fe_2(SO_4)_3$
The oxidation number of Fe in $FeSO_4$ = +2
The oxidation number of Fe in $Fe_2(SO_4)_3$ = +3
The n–factor of $FeSO_4$, $n_2 = 1 \times (3 - 2) = 1$
According to the law of equivalence,
(Equivalents of KMnO4 reacted) = (Equivalents of $FeSO_4$ reacted)
⇒ $n_1 \times M_1 \times V_1 = n_2 \times M_2 \times V_2$
⇒ $5 \times M_1 \times 10 = 1 \times 0.1 \times 10$
⇒ $M_1 = 0.02$ M
Molar mass of $KMnO_4$ = 158 gm/mol
⇒ Strength of $KMnO_4$ solution in grams per liter = 0.02 × 158 = 3.16 g/ℓ
⇒ Strength of $KMnO_4$ solution in grams per liter = 316 × 10^{-2} g/ℓ
⇒ Answer = 316

49. The possible reactions are:
$ZnSO_4 + BaS \rightarrow ZnS + BaSO_4$
$BaCO_3 + CH_3COOH \rightarrow (CH_3COO)_2Ba + H_2O + CO_2$

50. $25 \times M = 20 \times 1$
$M = \frac{20}{25} = \frac{4}{5} = 0.8$
$\Delta T_f = (i)(K_f)(m)$
$= (2)(2)\left(\frac{4}{5}\right) = \frac{16}{5} = 3.2$
Nearest Integer - 3

51.



Let incentre of the triangle be (α, β)

vertices of the traingle are $(\pm ae, 0)$ and $(acos\vartheta, bsin\vartheta)$

$$\alpha = \frac{(a\cos\vartheta)SS' + AS(-ae) + AS'(ae)}{SS' + AS + AS'} \quad \ldots(1)$$

Here $AS = a - ex_1 = a - eacos\vartheta = a(1 - ecos\vartheta)$
and $AS' = a + ex_1 = a + eacos\vartheta = a(1 + ecos\vartheta)$
and $SS' = 2ae$
Putting all three values in (1) we get

$$\Rightarrow \frac{2a^2 e\cos\vartheta[1 + e]}{2a(1 + e)} = \alpha$$

Similarly $\frac{2a^2 e\sin\vartheta}{2a + 2ae} = \beta$

$$\frac{ae\sin\vartheta}{1 + e} = \beta$$

Eliminating $cos\vartheta$ & $\sin\vartheta$ from values of α and β, we get

$$\frac{\alpha^2}{e^2} + \frac{\beta^2}{\left(\frac{e}{1 + e}\right)^2} = a^2$$

or locus $\frac{x^2}{e^2} + \frac{y^2}{\left(\frac{e}{1+e}\right)^2} = a^2$

where $\frac{e^2}{1 + e^2} = e^2(1 - e'^2)$

$e' = \frac{e}{\sqrt{1 + e^2}} = \frac{3}{5}$, where e' is eccentricity of obtained locus

52. $BD = S - b : CD = S - c$

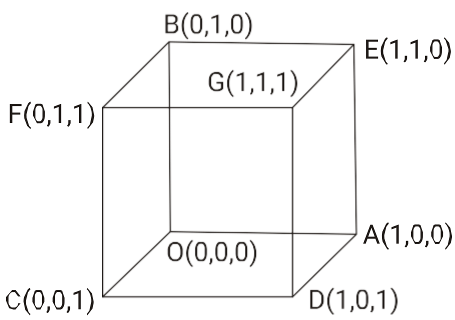
$(S - b)(S - c) = 2 \Rightarrow \Delta^2 = 2S(S - a)$

$\therefore \frac{\Delta^2}{S^2} = \frac{2(S - a)}{S} = 1 \Rightarrow 2 - \frac{2a}{S} = 1 \Rightarrow \frac{2a}{S} = 1$

$\Delta = \frac{1}{2} a h_a$; $\frac{\Delta}{S} = \frac{1}{2}\frac{a}{S} h_a = 1 \Rightarrow h_a = 4$

⇒ Locus of A lie on a straight line parallel to BC.

53.



DR'S of OG = 1, 1, 1
DR'S of AF = −1, 1, 1
DR'S of CE = 1, 1, −1
DR'S of BD = 1, −1, 1

Equation of $OG \Rightarrow \frac{x}{1} = \frac{y}{1} = \frac{z}{1}$

Equation of $AB \Rightarrow \frac{x - 1}{1} = \frac{y}{-1} = \frac{z}{0}$

Normal to both the line's

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = \hat{i} + \hat{j} - 2\hat{k}$$

$\vec{OA} = i$

$$\frac{|\hat{i}.(\hat{i} + \hat{j} - 2\hat{k})|}{|\hat{i} + \hat{j} - 2\hat{k}|} = \frac{1}{\sqrt{6}}$$

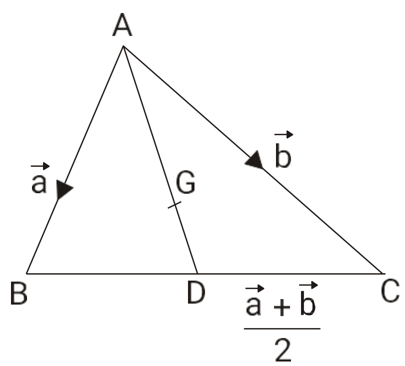
54. $\log_{(e+\pi)}(\log_2(\sqrt{4x + 1} + \sqrt{4x})) = 0$

or $\log_2(\sqrt{4x + 1} + \sqrt{4x}) = (e + \pi)^0$
or $\log_2(\sqrt{4x + 1} + \sqrt{4x}) = 1$
or $\sqrt{4x + 1} + \sqrt{4x} = 2$
or $\sqrt{4x + 1} = 2 - \sqrt{4x}$
On Squaring, we get
$4x + 1 = 4 + 4x - 4\sqrt{4x}$
or $4\sqrt{4x} = 3$
or $64x = 9$

55. First, we assume there are 12 places
and we have to arrange these 12 letters in these 12 places.
We choose 7 places out of 12 places
and arrange I I P P A Y U in these places.
Then there will only be 1 arrangement for the rest of the places
in this order R H H H R as all H's must lie between both R only.
$(HHH), (RR), {}_{V}(II), (P\,P), AYU_{S}$
Hence the number of arrangements

$$= {}^{12}C_7 . \frac{\lfloor 7}{\lfloor 2 . \lfloor 2} . 1 = (198)7!$$

56.



$$\vec{AG} = \frac{2}{3}\vec{AD}$$

$= \frac{2}{3}\left(\frac{\vec{a}+\vec{b}}{2}\right) = \frac{\vec{a}+\vec{b}}{3}$

57. $I = \int_0^\infty \frac{\ln x}{(x+3)^2} dx$

Put $x = 3t$

$I = 3\int_0^\infty \frac{\ln 3 + \ln t}{9(t+1)^2} dt$

Put $t = \frac{1}{u}$ and add

$\Rightarrow 2I = \frac{2}{3}\int_0^\infty \frac{\ln 3}{(t+1)^2} dt$

$\Rightarrow I = \frac{\ln 3}{3}$

58. $a \sin^2\vartheta + b\cos^2 = m$ ..(1)
$b\sin^2\phi + a\cos^2\phi = n$ ..(2)
$a\tan\vartheta = b\tan\vartheta$ (ii) ..(3)
Divide (1) by $\cos^2\vartheta$ we, get
$a\tan^2\vartheta + b = m\sec^2\vartheta$
$\Rightarrow \tan^2\vartheta = \frac{m-b}{a-m}$ ..(4)

Divide (2) by $\cos^2\phi$, we get
$B\tan^2\phi + a = n\sec^2\phi$
$\Rightarrow \tan^2\phi = \frac{n-a}{b-n}$ ..(5)

From (3), (4), (5)
$a^2\left(\frac{m-b}{a-m}\right) = b^2\left(\frac{n-a}{b-n}\right)$
$\Rightarrow a^2(mb - mn - b^2 + bn) = b^2(an - a^2 - mn - am)$
$\Rightarrow abm(a-b) + abn(a-b) = mn(a^2 - b^2)$
$\Rightarrow abm + abn = mn(a+b)$
Divide both sides by $abmn$, we get
$\frac{1}{n} + \frac{1}{m} = \frac{1}{a} + \frac{1}{b}$

59. $f(x) = \begin{cases} xe^{ax}; & x \le 0 \\ x + ax^2 - x^3; & x > 0 \end{cases}$

[$f(x)$ is continuous at $x = 0$]

$\Rightarrow f'(x) = \begin{cases} axe^{ax} + e^{ax}; & x \le 0 \\ 1 + 2ax - 3x^2; & x > 0 \end{cases}$

[$f'(x)$ is continuous at $x = 0$] and

$f''(x) = \begin{cases} 2ae^{ax} + a^2xe^{ax}; & x \le 0 \\ 2a - 6x; & x > 0 \end{cases}$

[$f''(x)$ is continuous at $x = 0$]
Now $f''(x) > 0 \Rightarrow 2a - 6x > 0$ if $x > 0$ or
$2ae^{ax} + a^2xe^{ax} > 0$ if $x \le 0$
$\Rightarrow x < \frac{a}{3}$ if $x > 0$ or $x > -\frac{2}{a}$ if $x \le 0$
$\Rightarrow f''(x) > 0$ if $-\frac{2}{a} < x < \frac{a}{3}$
$\therefore f'(x)$ increase if $x \in \left(-\frac{2}{a}, \frac{a}{3}\right)$

60. $\because a, b, c$ are in G.P.
$\therefore \frac{b}{a} = \frac{c}{b} = r$
$\Rightarrow \frac{b^2}{a^2} = \frac{c^2}{b^2} = r^2$
$\Rightarrow a^2, b^2, c^2$ are in G.P.

61. Given that
$P(A) = 0.4$..............(i)

Probability of negation of event A is
$P(A) = 1 - P(A)$
from (i)
$\Rightarrow P(A) = 1 - 0.4$
$\Rightarrow P(A) = 0.6$
Now the probability that the event A does not happen in any of the three independent trials
$= P(A).P(A).P(A) = (0.6)^3$
Thus the required probability
$= 1 - (0.6)^3$
$= 1 - 0.216$
$= 0.784$

62. $\pi n\left(1 + \frac{1}{n}\right)^{1/2} = n\pi\left(1 + \frac{1}{2n} + \frac{1}{2}\left(\frac{1}{2} - 1\right)\frac{1}{2!}\frac{1}{n^2} + \ldots\ldots\ldots\right)$
$= \left[n + \frac{1}{2} + \frac{1}{2}\left(\frac{1}{2} - 1\right)\frac{1}{2!}\frac{1}{n} + \ldots\ldots\ldots\right)$
As
$n \to \infty;\ \frac{\pi}{2}\cdot\left(2n + 1 + \left(\frac{1}{2} - 1\right)\frac{1}{2!}\frac{1}{n} + \ldots\ldots\ldots\right) = (2n+1)\frac{\pi}{2}$

alternatively (1) Best
$I = \underset{n\to\infty}{Limit} \pm \cos(n\pi - \pi\sqrt{n^2 + n})$

$\Rightarrow \underset{n\to\infty}{Limit} \pm \cos(\pi(n - \sqrt{n^2 + n}))$
On rationalising $\ell = \underset{n\to\infty}{Limit}\cos\left(\frac{(\pi(-n))}{n + \sqrt{n^2 + n}}\right)$

$\underset{n\to\infty}{Limit}\cos\left(\frac{n\pi}{n + n\sqrt{1 + \frac{1}{n}}}\right)$

$= \underset{n\to\infty}{Limit}\cos\left(\frac{\pi}{1 + \sqrt{1 + \frac{1}{n}}}\right) = \cos\frac{\pi}{2} \to 0$

63. Let the point P be (h, k)
$\Rightarrow m^3 + (2 - h)m + k = 0$
$s_1 = 0;\ s_2 = 2;\ s_3 = -k$
$\tan\alpha_1 = \frac{s_1 - s_3}{1 - s_2} = \frac{k}{h-1}$ and $\tan\alpha_2 = \frac{k}{h-1} = m_{PS}$
$\Rightarrow \alpha_1 - \alpha_2 = n\pi$
$\Rightarrow \frac{\alpha_1 - \alpha_2}{\pi}$ is an integer

64. Combined equation of pair of lines joining origin and point of intersection of circle and line

$x^2 + y^2 = 10\left(\frac{\sqrt{5}x + 2y}{3\sqrt{5}}\right)^2$

$\frac{1}{9}x^2 - \frac{1}{9}y^2 + \frac{8}{9}\sqrt{5}xy = 0$

$a + b = \frac{1}{9} - \frac{1}{9} = 0$

$\therefore$ Δ is right angled triangle at origin $(0, 0)$

$\therefore$ Area $= \frac{1}{2}r^2 = \frac{1}{2}(\sqrt{10})^2 = 5$

65. Let $f(x) = x\sin x + \cos x$

$\Rightarrow f'(x) = x\cos x$

$\Rightarrow f''(x) = -x\sin x + \cos x$

Now $I = \int e^{x\sin x+\cos x}\left(\frac{x^4\cos^3 x - x\sin x + \cos x}{x^2\cos^2 x}\right)dx$

$= \int e^{x\sin x+\cos x}\left(\frac{x^4\cos^3 x}{x^2\cos^2 x} + \frac{-x\sin x + \cos x}{x^2\cos^2 x}\right)dx$

$= \int e^{x\sin x+\cos x}\left(x\cdot x\cos x + \frac{-x\sin x + \cos x}{x^2\cos^2 x}\right)dx$

Which can be expressed as

$\int e^{f(x)}\left(xf'(x) + \frac{f''(x)}{(f'(x)^2)}\right)dx$

$= \int xe^{f(x)}f'(x)dx + \int e^{f(x)}\cdot\frac{f''(x)}{(f'(x))^2}dx$

$= xe^{f(x)} - \int e^{f(x)}dx + e^{f(x)}\frac{-1}{f'(x)} - \int e^{f(x)}\cdot f'(x)\left(\frac{-1}{f'(x)}\right)dx$

$= xe^{f(x)} - \frac{e^{f(x)}}{f'(x)} + C$

$= e^{f(x)}\left(x - \frac{1}{f'(x)}\right) + C$

$= e^{x\sin x+\cos x}\left(x - \frac{1}{x\cos x}\right) + C$

66. $\frac{dy}{dx} + \frac{x + a}{y - 2} = 0$

$(y - 2)dy + (x + a)dx = 0$

Integrating

$\frac{y^2}{2} - 2y + \frac{x^2}{2} + ax = C$

Or $x^2 + 2ax + y^2 - 4y = C$

At $x = 1, y = 0$

$1 + 2a = C$

Equation of circle

$x^2 + 2ax + y^2 - 4y = 1 + 2a$

$x^2 + y^2 + 2ax - 4y - (1 + 2a) = 0$

$r = \sqrt{a^2 + 4 + 1 + 2a} = 2$

$a^2 + 2a + 5 = 4 \Rightarrow a = -1$

Curve is $x^2 + y^2 - 2x - 4y + 1 = 0$

Intersection with y–axis

$P = (0, 2 + \sqrt{3})$ $Q \equiv (0, 2 - \sqrt{3})$

For normal at P & Q

$R\left(1 + \frac{2}{\sqrt{3}}, 0\right), S = \left(1 - \frac{2}{\sqrt{3}}, 0\right)$

$RS = \frac{4\sqrt{3}}{3}$

67. $f(x) = \sum_{k=1}^{n}\left(1 + \left[\sin\left(\frac{kx}{n}\right)\right]\right)$

$= n + [\sin\frac{x}{n}] + [\sin\frac{2x}{n}] + \ldots + [\sin x]$ .... (i)

Case I : When $x \neq \frac{\pi}{2}$

Since $0 < \frac{kx}{n} < \pi$ and

$\frac{kx}{n} \neq \frac{\pi}{2}$ $\therefore 0 < \sin\left(\frac{kx}{n}\right) < 1$, for $k = 1, 2, \ldots, n$

$\therefore \left[\sin\left(\frac{kx}{n}\right)\right] = 0$, for $k = 1, 2, 3, \ldots, n$

$\therefore$ From (i), $f(x) = n$

Case I : When $x = \frac{\pi}{2}$

In this case $\sin x = 1$ and others lie between 0 and 1.

$\therefore$ From (i), $f(x) = n + 1$.

Hence range of $f = \{n, n + 1\}$.

68. Given A and B are orthogonal matrix, which gives

$AA^T = I = BB^T$

Considering $AA^T = I$

$\Rightarrow |AA^T| = |I|$

$\Rightarrow |A||A^T| = 1$

We know $|A| = |A^T|$

$\Rightarrow |A|^2 = 1$

$\Rightarrow |A| = \pm 1$

If $|A| = 1$, then $|B| = -1$ $\{\because |A| + |B| = 0\}$

Or

If $|A| = -1$, then $|B| = 1$ $\{\because |A| + |B| = 0\}$

Consider

$|A^T(A + B)B^T|$

$= |A^TAB^T + A^TBB^T|$

$= |B^T + A^T|$

$= |(A + B)^T|$

$= |A + B|$ ..(i)

Also $|A^T(A + B)B^T|$

$= |A^T||A + B||B^T|$

$= |A||A + B||B|$

$= = -|A + B|$ $\{$As $|A| = 1, |B| = -1\}$ ..(ii)

From (i) and (ii), we have

$|A + B| = -|A + B|$

$\Rightarrow 2|A + B| = 0 \Rightarrow |A + B| = 0$

69. Let $u = \frac{z - 1}{e^{\vartheta i}} \Rightarrow \frac{e^{\vartheta i}}{z - 1} = \frac{1}{u}$. Now

$\left(u + \frac{1}{u}\right) - \left(u + \frac{1}{u}\right) = 0 \Rightarrow (u - u)\left(1 - \frac{1}{uu}\right) = 0$

If u is not purely real, then

$uu = 1 \Rightarrow \frac{z - 1}{e^{\vartheta i}} = 1 \Rightarrow |z - 1| = 1$

70. Given $2x = y^{1/5} + y^{-1/5}$,

We know, $(y^{1/5} - y^{-1/5})^2 = (y^{1/5} + y^{-1/5})^2 - 4$

$\Rightarrow (y^{1/5} - y^{-1/5})^2 = 4x^2 - 4$ [using $2x = y^{1/5} + y^{-1/5}$]

$\therefore y^{1/5} - y^{-1/5} = \pm 2\sqrt{x^2 - 1}$ .... (i)

Also $y^{1/5} + y^{-1/5} = 2x$ ......(ii) [Given]

Adding (i) and (ii), we get

$2y^{1/5} = 2x \pm 2\sqrt{x^2 - 1}$

$\therefore\ y^{1/5} = x \pm \sqrt{x^2 - 1}$

Or $y = (x \pm \sqrt{x^2 - 1})^5$ ..(iii)

On differentiating both sides w.r.t. x, we get

$\frac{dy}{dx} = 5(x \pm \sqrt{x^2 - 1})^4 . \{1 \pm \frac{1.2x}{2\sqrt{x^2 - 1}}\}$

Using (+) sign, we get

$\frac{dy}{dx} = \frac{5(x + \sqrt{x^2 - 1})^5}{\sqrt{x^2 - 1}} = \frac{5y}{\sqrt{x^2 - 1}}$

Or $(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 25y^2$

Using (-) sign, we get

$\frac{dy}{dx} = \frac{5(x - \sqrt{x^2 - 1})^5}{\sqrt{x^2 - 1}} = -\frac{5y}{\sqrt{x^2 - 1}}$

Or $(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 25y^2$ ..(iv)

Again differentiating both sides w.r.t. x, we get,

$2x\left(\frac{dy}{dx}\right)^2 + (x^2 - 1)2\frac{dy}{dx}.\frac{d^2y}{dx^2} = 50y\frac{dy}{dx}$,

Dividing by $2\frac{dy}{dx}$ on both sides, we get

$x\frac{dy}{dx} + (x^2 - 1)\frac{d^2y}{dx^2} = 25y \ \{\because \frac{dy}{dx} \neq 0\}$

71. $\left(7^{\left(\frac{1}{2}\right)} + 11^{\left(\frac{1}{6}\right)}\right)^{824}$

Number of integral term $T_{r+1} = {}^{824}C_r\left(7^{\frac{1}{2}}\right)^{824-r}\left(11^{\frac{1}{6}}\right)^r$

$\Rightarrow r$ must be multiple of 6

$\Rightarrow r = 0, 6, 12, \ldots\ldots 822$

$\Rightarrow 138$ term

72. Let $\alpha = n^2, \beta = (n + 1)^2$

$\sqrt{\alpha\beta} = |\alpha - \beta| + 1$

Hence, $n = 2$

$\therefore \alpha = 4, \beta = 9$

Now, $f(4) \cdot f(9) < 0$ ans also checking boundary points.

We get $k \in [12, \frac{113}{9}]$.

73. $a_1 + a_2 + a_3 + \ldots + a_7 = 9k, k \in I$.

Also $a_1 + a_2 + \ldots + a_9 = 1 + 2 + 5 + \ldots + 9 = 45$

$\Rightarrow a_8 + a_9 = 45 - 9k$

$\Rightarrow 3 \le a_8 + a_9 \le 17$

$\Rightarrow k = 4$

$\Rightarrow a_8 + a_9 = 9$

$\Rightarrow (1, 8)(2, 7)(3, 6)(4, 5)$

$P(E) = \frac{4}{36} = \frac{1}{9}$.

74. For P_n

First, line up the $n - 3$ people not selected and then choose 3 of the $n - 2$ gaps they create

$\Rightarrow P_n = {}^{n-2}C_3$

For Q_n

First, assume that the n people are in a row

Then there are ${}^{n-2}C_3$ ways to select 3 people so that no tow are consecutive.

Now we must subtract the $n - 4$ possiblities where the two people gaps at the end were choosen

$\Rightarrow Q_n = {}^{n-2}C_3 - (n - 4)$

Now

$P_n - Q_n = {}^{n-2}C_3 - ({}^{n-2}C_3 - (n - 4))$

$= n - 4 \Rightarrow n - 4 = 6$

$\Rightarrow n = 10$

75. Let $f(x) = x^4 \cdot 3^{|x-2|} \cdot 2^{|x-5|} \cdot 5^{x-1}$

Now, $|f(x)| = -f(x)$ (Given)

$\Rightarrow f(x) \le 0$

$\Rightarrow x^4 \cdot 3^{|x-2|} \cdot 2^{|x-5|} \cdot 5^{x-1} \le 0$,

Which is only possible when $x^4 = 0$

$\therefore x = 0$