1. $U_{in} = \frac{1}{2}\frac{Q^2}{C}$

On halving the distance $C' = \frac{C}{2}$; Q = constant

$$U_{final} = \frac{1}{2}\frac{Q^2}{C'} = \frac{1}{2}\frac{Q^2}{\left(\frac{C}{2}\right)} = \frac{Q^2}{C}$$

Hence work done

$$W = U_{final} - U_{in} = \frac{Q^2}{C} - \frac{1}{2}\frac{Q^2}{C} = \frac{Q^2}{2C}$$

2. Magnetic field induction at point inside the solenoid of length l, having n turns per unit length carrying current i is given by $B = \mu_0 ni$.

Given that i is doubled and number of turns per unit length are halved.

$B' = \mu_0 \frac{n}{2}(2i) = \mu_0 ni = B$

3. Given that u and v are functions of x that are differentiable at x = 0 and u(0) = 5, u′(0) = −3, v(0) = −1 and v′(0) = 2, where u′ and v′ are the derivatives of u and v with respect to x. We have to find $\frac{d}{dx}(uv)$ at x = 0.

Using product rule, we get, $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$.

$\rightarrow \frac{d}{dx}(uv) = uv' + vu'$

So, $\frac{d}{dx}(uv)\Big|_{x=0} = u(0)v'(0) + v(0)u'(0)$

$\rightarrow \frac{d}{dx}(uv)\Big|_{x=0} = 5(2) + (-1)(-3) = 13$

4. Given, wavelength of waves is $\lambda = 4$ m.

Intensity of each source $I_1 = I_2 = I_o$.

Resultant intensity at P is, $I_{res} = 2I_o$.

Distance between the source S_1 and point P = $S_1P = X$.

We know that, the resultant intensity $I_{res} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$, where ϕ is phase difference between the sources.

$\rightarrow 2I_o = I_o + I_o + 2\sqrt{I_o \times I_o}\cos\phi$

$\rightarrow \cos\phi = 0$

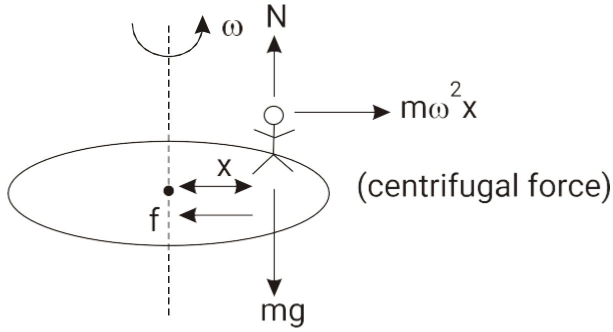
$\rightarrow \phi = \frac{n\pi}{2} = \frac{\pi}{2}$ (n = 1 for S_2P to be minimum)

We know that, the relation between the phase difference (ϕ) and path difference (Δx) is $\phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x)$

$\rightarrow \phi = \left(\frac{2\pi}{4}\right)\Delta x = \frac{\pi}{2}$

5. Given that a boy is sitting on the horizontal platform of a joy wheel at a distance of 5 m from the center. The wheel begins to rotate and when the angular speed exceeds 1 rad/s, the boy just slips. We have to find the coefficient of friction between the boy and the wheel. (g = 10 m/s^2)

The FBD of the boy in the frame of reference of the joy wheel is shown below. It is a non-inertial frame of reference. So, a centrifugal force acts on the boy that acts radially outwards.



Friction (f) acts radially inwards to balance the centrifugal force.
The boy slips when the maximum friction on the boy is just able to balance the centrifugal force.

$\rightarrow \mu N = m\omega^2 x$.

Now, there is no motion in the vertical direction. So, we get, N = mg.

$\rightarrow \mu mg = m\omega^2 x$

$\rightarrow \mu = \frac{\omega^2 x}{g}$

$\rightarrow \mu = \frac{1^2 \times 5}{10} = 0.5$

6. $K_{max} = 0.5$ eV

$\rightarrow 0.5 = 2.48 - \phi$

$\rightarrow \phi = 1.98$ eV

7. Given,
The distance between the edges of the wings = 10 m.
The speed of the flight = 180 Km/h along the horizontal direction.
The total intensity of the earth's field at that part B = 2.5×10^{-4} Wb/m^2
The angle of the dip $\vartheta = 60°$.
So verticle component of magnetic field is, $B_v = B\sin\vartheta$.

We know that the motional emf induced across the rod

→ → →

$\rightarrow \Delta x = 1$ m

$\rightarrow \Delta x = S_2P - S_1P = 1$ m.

$\rightarrow S_2P = S_1P + 1 = (x + 1)$ meters.

is, $\varepsilon = L \cdot [V \times B]$

→ $\varepsilon = B_v VL \sin \vartheta$

→ $\varepsilon = BVL \sin \vartheta$

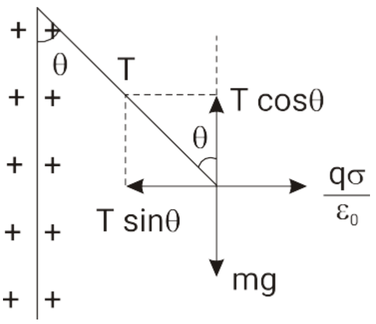
→ $\varepsilon = (2.5 \times 10^{-4} T) \left(180 \times \frac{5}{18} m/s\right) (10m) \sin 60^{\circ}$

→ $\varepsilon = 108.25 \times 10^{-3}$ V.

8. Given,
The mass of the sphere = m. The charge on the sphere = q.
The angle made by the silk thread with the vertical

conducting charged plate = ϑ.
We know that the electric field produced by the conducting plate is, E = $\frac{\sigma}{\epsilon_0}$ and is away from it.

The sphere gets repelled by the field of the plate and the electric force on it is, $F = qE = \frac{q\sigma}{\epsilon_0}$.
If we observe the force acting on the point charge then the free body diagram is as shown in the figure.



Let T be the tension in the thread.
For equilibrium conditions,

Along horizontal direction, $T\sin\vartheta = \frac{q\sigma}{\epsilon_0}$.

Along the vertical direction, $T\cos\vartheta = mg$.

Dividing these two equations, we get, $\frac{T\sin\vartheta}{T\cos\vartheta} = \left(\frac{\frac{q\sigma}{\epsilon_0}}{mg}\right)$.

$\rightarrow \tan\vartheta = \frac{q\sigma}{\epsilon_0 mg}$.

$\therefore \sigma = \frac{\epsilon_0 mg\tan\vartheta}{q}$.

9. We have, $a_{max} = A\omega^2$.

So, $\frac{a_{max}}{A} = \omega^2$.

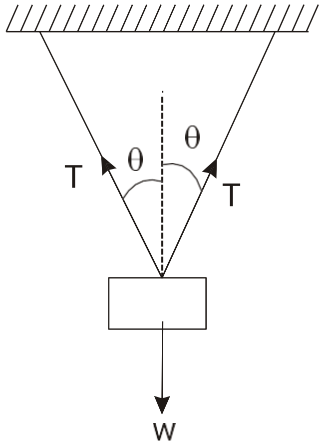
10. $E = -\frac{K}{r^2}$

$\Delta V = -\int_{r=2\text{ cm}}^{r=3\text{ cm}} E\,dr = \int_2^3 \frac{K}{r^2}\,dr$

$\rightarrow \Delta V = \left[-\frac{K}{r}\right]_2^3 = \left(\frac{K}{6}\right) = \frac{6}{6} = 1$ J/kg

$\rightarrow V_f - V_i = 1$
$\rightarrow V_f = 11$ J/kg

11. Given that a block of weight W is suspended by a string of fixed length. The ends of the string are held at various positions as shown in the figures in the options.



Because of symmetry, the tensions in the two strings will be equal.
The block is in equilibrium. So, the net force on it is zero.
For the vertical equilibrium of the block, we get,
$2T\cos\vartheta = W$
$\rightarrow T = \frac{W}{2\cos\vartheta}$

For A, ϑ is maximum among the four options.

So, $\cos\vartheta$ is minimum and hence T is maximum.

12. Given that, input voltage $V = V_0\sin\omega t + \frac{V_0}{2}\cos\omega t$

$\rightarrow V = V_0\sqrt{1 + \frac{1}{4}}\,\sin(\omega t + \alpha)$

$\rightarrow V = \frac{V_0}{2}\sqrt{5}\,\sin(\omega t + \alpha)$

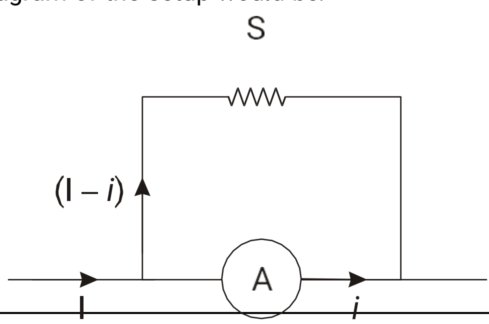
Current in an AC circuit can be written as $I = \frac{V}{Z}$, for LR circuit we can write $Z = \sqrt{(\omega L)^2 + R^2}$

So, amplitude of current $I_0 = \frac{V_0\sqrt{5}}{2\sqrt{\omega^2L^2 + R^2}}$

$\therefore I_0 = \sqrt{\frac{5V_0^2}{4(\omega^2L^2 + R^2)}}$

13. Given that the ammeter has a range of 1 ampere without shunt.

The diagram of the setup would be:



Potential difference across the shunt and ammeter would be the same.

$(I - i)S = ir$, where r is the resistance of ammeter without shunt.

On substituting maximum range of i we get, $S = \frac{1 \times r}{(I - 1)}$.

As we increase I the value of shunt decreases, this behaviour is showed by only graph Q.

14. Given that in a vessel, equal masses of alcohol of specific gravity 0.8 and water are mixed together. A capillary tube of radius 1 mm is dipped vertically in it. If the mixture rises to a height 5 cm in the capillary tube, we have to find the surface tension of the mixture (the contact angle is zero).

The height to which the liquid mixture rises in the capillary is, $h = \frac{2S\cos\vartheta}{\rho g r}$, where S = the surface tension of the mixture, ρ = the density of the mixture, r = the radius of the capillary and ϑ = the contact angle.

So, $h = \frac{2S\cos 0^\circ}{\rho g r}$.

$\rightarrow h = \frac{2S}{\rho g r}$ ....(i)

Let us find ρ, the density of the mixture.

Now, $\rho(v_1 + v_2) = 2m$, where v_1 is the volume of water, v_2 is the volume of alcohol and m is the mass of water and alcohol each.

Also, $v_1 = \frac{m}{\rho_1}$, where ρ_1 is the density of water and

$v_2 = \frac{m}{\rho_2}$, where ρ_2 is the density of alcohol.

$\rightarrow \rho\left(\frac{m}{\rho_1} + \frac{m}{\rho_2}\right) = 2m$

$\rightarrow \rho\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right) = 2$

$\rightarrow \rho = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}$

Now, $\rho_2 = 0.8\rho_1$.

$\rightarrow \rho_2 = 0.8(1) = 0.8$ g/cm^3

$\therefore \rho = \frac{2(1)(0.8)}{1 + 0.8} = \frac{8}{9}$ g/cm^3.

$\therefore$ From (i), we get, $5 = \frac{2S}{\left(\frac{8}{9}\right)(980)(0.1)}$. (We have everything in the CGS system here, because all the options are in CGS (dyne/cm).)

$\rightarrow S = 217.8$ dyne/cm

15. We have, $I_0 = 32$ W/m^2

After 1st polarizer, $I_1 = \frac{I_0}{2} = 16$ W/m^2.

Let the transmission axis of the 2nd polarizer be at an angle ϑ with that of the first one, then the 3rd one's pass axis is at an angle $90^\circ - \vartheta$ with the 2nd one because the 1st and the 3rd are at 90°.

So, $I_2 = 16\cos^2\vartheta$

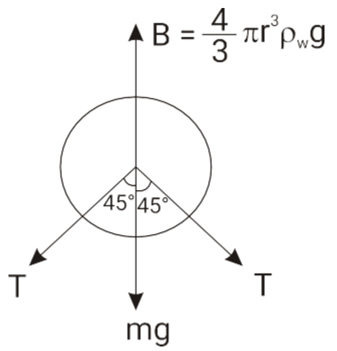
~~$\Rightarrow I_3 = (16\cos^2\vartheta)\cdot\cos^2(90 - \vartheta)$~~

$\rightarrow 16\cos^2\vartheta\sin^2\vartheta = 3$

$\rightarrow 4\sin^2 2\vartheta = 3$

$\rightarrow \sin 2\vartheta = \frac{\sqrt{3}}{2}$

The sphere is tied to the bottom of the tank by two wires making angles 45° with the horizontal.



As the sphere is in equilibrium, net force acting on the sphere is zero.

Upward buoyant force = $2T\cos 45^\circ$ + Weight

$\rightarrow \frac{4}{3}\pi R^3\rho_w g - mg = T\sqrt{2}$

$\rightarrow T = \frac{\frac{4}{3}\pi R^3\rho_w g - mg}{\sqrt{2}}$

17. Given that an uniform, thin cylindrical shell and solid cylinder roll horizontally without slipping. The initial speed of the cylindrical shell = v and of the solid cylinder = v'.

The solid cylinder and the hollow cylinder encounter an incline that they climb without slipping. The maximum height they reach is the same.

As frictional force (point of contact does not slip) and normal reaction does not do any work, mechanical energy of bodies is conserved during the motion.

For cylindrical shell (M.E conservation),

$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + 0 = mgh + 0 + 0$

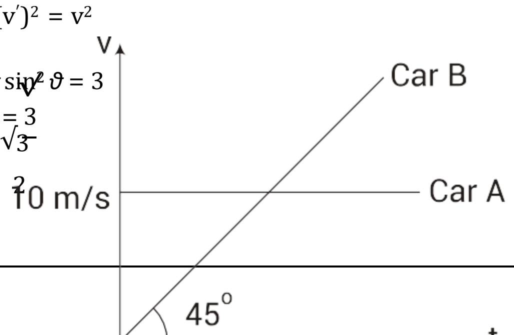
$\rightarrow \frac{1}{2}mv^2 + \frac{1}{2}(mR^2)\frac{v^2}{R^2} = mgh$, where K is radius of Gyration.

$\rightarrow v^2 = gh$ ..... (1)

Similarly for solid cylinder, $(v')^2\left(1 + \frac{1}{2}\right) = 2gh$ ...... (2)

From equations (1) and (2), $2v^2 = (v')^2\left(1 + \frac{1}{2}\right)$

$\rightarrow \frac{3}{4}(v')^2 = v^2$



$\rightarrow v'$ $\overline{}$ $=$ $\underline{4}$

$\rightarrow 2\vartheta = 60°$

$\rightarrow \vartheta = 30°$

16. A hollow sphere of mass M and radius R is immersed in a tank of water (density ρ_w). The sphere would float if it were set free.

v. 3

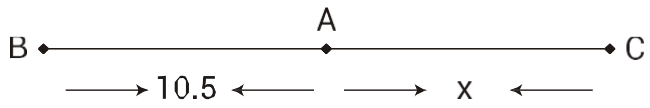
18. Given that intially car A is 10.5 m ahead of car B.
The velocity-time graph of two cars is shown in figure.

From graph we can say, $u_A = 10$ m/s and $u_B = 0$ m/s.

We know that, slope of velocity (vs) time graph is acceleration.

→ $a_A = 0$ and $a_B = \tan 45^\circ = 1$ m/s^2.

Let us assume that they meet at C after time t.



Apply the basic equation of kinematics $s = ut + \frac{1}{2}at^2$ for car A and car B.

For car A : $x = u_A t$

→ $x = 10t$

For car B, $x + 10.5 = \frac{1}{2}(a_B t^2)$

→ $10t + 10.5 = \frac{1}{2} \times 1 \times t^2$

→ $t^2 - 20t - 21 = 0$

→ $(t - 21)(t + 1) = 0$

→ $t = 21$ sec

Note: Relative motion concept also can be used.

19. Given that power applied to a particle varies with time as $P = (3t^2 - 2t + 1)$ watt, where t is in second.
Power applied to particle refers to rate at which work is done on the particle.

So $\frac{dW}{dt} = 3t^2 - 2t + 1$

→ $\int_2^4 dW = \int_2^4 (3t^2 - 2t + 1)dt$

→ $W_{(t=2 \text{ to } t=4)} = (t^3 - t^2 + t)_2^4$

→ $W_{(t=2 \text{ to } t=4)} = (64 - 16 + 4) - (8 - 4 + 2)$

→ $W_{(t=2 \text{ to } t=4)} = 46$ J

For a single particle system, net work done on a particle is equal to the change in kinetic energy of the particle. So change is kinetic energy of the particle from t = 2 sec to t = 4 sec is 46 J

20. For lens 1: $f_1 = 10, u = -30, v = ?$

$v = \frac{uf}{u+f} = \frac{-30 \times 10}{-30 + 10} = 15$ cm

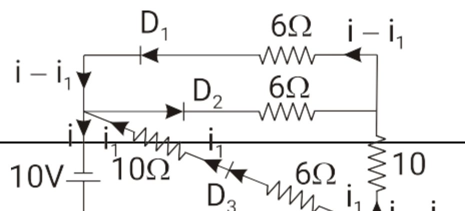
For lens 2: $f_2 = -10, u = 10, v = ?$

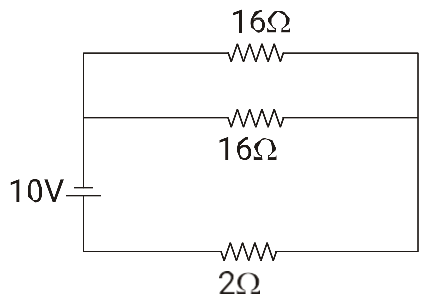
$v = \frac{uf}{u+f} = \frac{10 \times (-10)}{10 - 10} = \infty$

For lens 3: $f = 30, u = -\infty, v = ?$

So v will be 30 cm.

21.





$V = IR_{net}$

$10 = I \times 10$

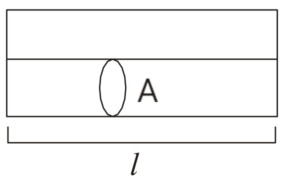
$I = 1A$

Ans. 1

22. $\beta = \frac{\lambda D}{d} = \frac{5 \times 10^{-7} \times 2}{3 \times 10^{-4}} = \frac{10 \times 10^{-3}}{3}$

For 3rd maximum, $x = 3\beta = 10 \times 10^{-3}$ m = 10 mm

23. Given that two wires of same length and thickness having specific resistances 6 Ω-cm and 3 Ω-cm respectively are connected in parallel.
Let the length of each wire = l, cross-sectional area of each wire = A.



When they are placed in parallel, the length of the effective resistance = l.

The cross-sectional area of the effective resistance = 2A.

Consider the resistivity of effective resistance = ρ.
We know that when the resistances are connected in parallel, the effective resistance is given by

$R_{net} = \frac{R_1 R_2}{R_1 + R_2}$.

→ $\frac{\rho l}{2A} = \frac{\rho_1 \frac{l}{A} \times \rho_2 \frac{l}{A}}{\rho_1 \frac{l}{A} + \rho_2 \frac{l}{A}}$.

→ $\frac{\rho}{2} = \frac{\rho_1 \rho_2}{\rho_1 + \rho_2}$

→ $\frac{\rho}{2} = \frac{6 \times 3}{6 + 3} = 2$.

→ $\rho = 4$ Ω-cm.

24. The vector $\vec{C} = 5\left(\frac{4\hat{i} + 3\hat{j}}{5}\right) = 4\hat{i} + 3\hat{j}$

x-component = 4

25. e^- should have a velocity away from the sheet at t = 0.

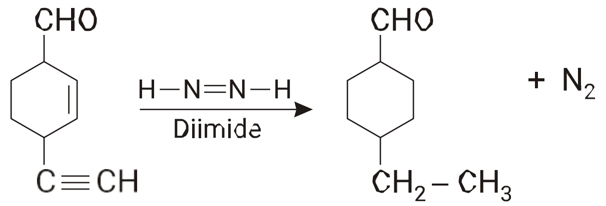
$\because s = ut + \frac{1}{2}at^2$

$-1 = 1(1) + \frac{1}{2}\left(\frac{-\sigma e}{2\varepsilon_0 m}\right)(1)^2$

$\therefore \sigma = \frac{8\varepsilon_0 m}{e}$

26. The gap between radii of 4d and 5d elements is very less due to lanthanide contraction and size of 5d is greater than size of 4d.

27. Diimide (N_2H_2) is specifically used for hydrogenation of non-polar multiple bonds like C=C, C≡C, N=N, etc.
The advantage of using diimide is that it does not react with polar bonds like C=O, C=N, C≡N, etc.
→ The diamide won't affect the aldehyde group present in the compound.



→ Option (A) is ***CORRECT.***

28. $CH_3-\overset{O}{\overset{\|}{C}}-CH_3 \xrightarrow{[O]} CH_3COOH + HCOOH$

$CCl_3CH_2CHO \xrightarrow{[0]} CCl_3CH_2COOH$

$CH_3CH_2CH_2OH \xrightarrow{[0]} CH_3CH_2COOH$

$CH_3CH_2CHO \xrightarrow{[0]} CH_3CH_2COOH$

29. Potential Energy: P.E.
Kinetick Energy: K.E.
According to Bohr's model, the relation between P.E. and K.E. is:
P.E. = −2 × K.E.
→ K.E. = $\frac{1}{2}$ × P.E.
Given, P.E. of H atom = x eV
K.E. = $\frac{1}{2}$ × P.E. = $\frac{x}{2}$ eV

30. A reaction is said to be in equilibrium when the rate of forward reaction is equal to the rate of backward reaction.

31. Second of thermodynamics implies that heat cannot spontaneously flow from cold regions to hot regions without external work being performed on the system, which is evident from ordinary experience of refrigeration.
For example, in a refrigerator, heat flows from cold (inside of refrigerator) to hot (room), but only when forced by an external agent, the refrigeration system or electrical energy.

32. 3° alcohols are resistant to oxidation under drastic condition. They first form ketone and then acid by loosing one carbon at each step.

$$\underset{(4C)}{3^\circ\text{alcohol}} \xrightarrow[\text{[O]}]{\text{Drastic condition}} \underset{(3C)}{\text{ketone}} \xrightarrow{[O]} \underset{(2C)}{\text{acid}}$$

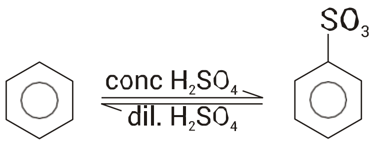
∴ Acid having 2C is formed when 3° alcohol is oxidised under drastic conditions.

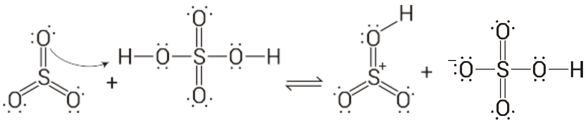
33. 1° and 2° amines only react with p-toluenesulfonyl chloride and the product of 1° amine only is soluble in

[ΔH = -ve → exothermic reaction]

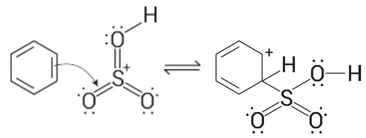
35. The combination of thermodynamic and kinetic factors leads to the stability of oxides of halogens in the following order:
I > Cl > Br
→ Option (D) is ***CORRECT.***

36. Sulphonation is reversible and it show kinetic isotopic effect.

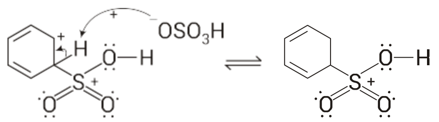


Step 1 

SO_3 is protonated to form SO_3H^+.

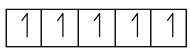
Step 2 

SO_3H^+ reacts as an electrophile with the benzene ring to form an arenium ion.

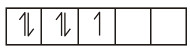
Step 3 

Loss a proton form the arenium ion restores aromaticity to the ring and the regenrates the acid catalyst.

37. In $[Fe(H_2O)_6]^{3+} \rightarrow Fe^{3+} \rightarrow 3d^5 \rightarrow$



(H_2O is moderate field ligand)
In $[Fe(CN)_6]^{3+} \rightarrow Fe^{3+} \rightarrow 3d^5 \rightarrow$



Different number of unpaired electrons, different magnetic behaviour.

38. $6HCHO + 4NH_3 \longrightarrow (CH_2)_6N_4$

39. Se^{2-} have maximum size due to less effective nuclear

aqueous NaOH. (Hinsberg test)

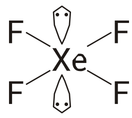
34. $\Delta H = E_a$ (forward) - E_a (backward) < 0

$[E_a < E_b]$

charge.

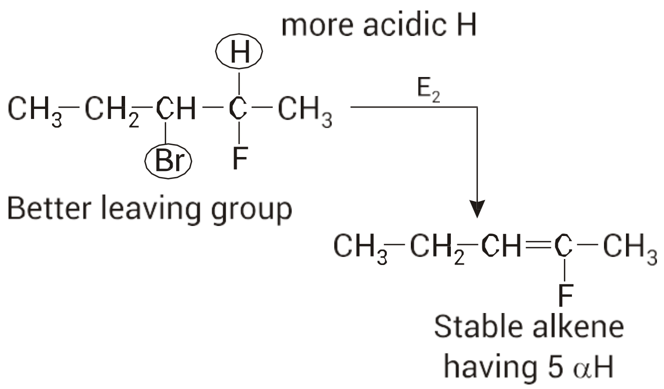
40. Osmosis involves movement of solvent from (1) High vapour pressure to low vapour pressure (2) Low

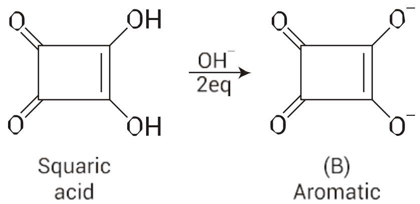
osmotic pressure to high osmotic pressutre and low concenteration of solution to high concenteration of solution.

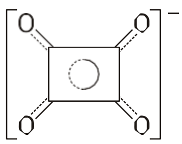
41. 

→ There are 4 bond pairs and 2 lone pairs.
→ The hybridization of Xe in XeF_4 is sp^3d^2

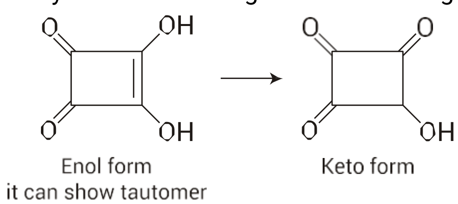
42. As —Br is a better leaving group and, due to –I effect of Flourine atom hydrogen present on the carbon-containing fluorine will be most acidic.
So, the product formed will be by elimination of Bromine.



43. 



Resonance hybrid of B showing all C-C bond length same



44. The given 200 gm sample of $CaCO_3$ is only 50% pure.
Thus, the mass of pure $CaCO_3$ available for the reaction = 50% of 200 gm = 100 gm
Hence, moles of $CaCO_3$ available for the reaction = $\frac{\text{mass}}{M.W.} = \frac{100}{100} = 1$

The decomposition reaction of $CaCO_3$ is

$$CaCO_3 \xrightarrow{\Delta} CaO + CO_2 \uparrow$$

From the above reaction:

moles of CO_2 produced = moles of $CaCO_3$ used

→ moles of CO_2 produced = 1

Thus, volume occupied at S.T.P by the CO_2 produced = moles of CO_2 produced × 22.4 L

→ Volume of evolved CO_2 at STP= 1 × 22.4 = 22.4 L

45. $BaCl_2$, NaCl are soluble but on adding HCl(g) to $BaCl_2$, NaCl solutions, Sodium or Barium chlorides may precipitate out, as a consequence of the law of mass action.

46. Given:
The change in free energy,
$\Delta G = -49.4\ \text{KJ/mol} = -49500\ \text{J/mol}$
The enthalpy change for the reaction,
$\Delta H = 51.4\ \text{KJ/mol} = 51400\ \text{J/mol}$
Using the equation for free energy change we get,
$\Delta G = \Delta H - T\Delta S$
→ – 49400 = 51400 – 300 ×ΔS
→$\Delta S = \frac{+100800}{300} = 336\ JK^{-1}mol^{-1}$
→ Answer = 336

47. True statements : (i), (iii), (iv), (vii).

48. Number of electrons in single N atom = 7
→ Number of electrons in single N^{3-} ion = (7 + 3) = 10
→ Number of moles of electrons in 1 mole of N^{3-} ion = 10 moles
→ Number of moles electrons in 0.5 mole of N^{3-} ion = 0.5 x 10 moles = 5 moles
→ Answer = 5

49. $R_f = \frac{\text{Distan ce travelled by the substance}}{\text{Distance travelled by the solvent front}}$
$(R_f)_A = \frac{2.08}{3.25}$
$(R_f)_B\ \frac{1.05}{3.25}$
$\frac{(R_f)_A}{(R_f)_B} \simeq 2$

50. Sulphur exhibits a highest oxidation state of +6 in sulphate (SO_4^{2-}). Conc. HNO_3 oxidises sulphur to sulphate and in this process, will get reduced to NO_2.
Oxidation Half Reaction:
$32H_2O + S_8 \rightarrow 8H_2SO_4 + 48H^+ + 48e^-$
Reduction Half Reaction:
$e^- + H^+ + HNO_3 \rightarrow NO_2 + H_2O$
We get the overall reaction by multiplying the reduction half reaction by 48 and adding it with the oxidation half reaction.
The overall reaction is:
$S_8 + 48HNO_3 \rightarrow 8H_2SO_4 + 48NO_2 + 16H_2O$
According to the stoichiometry of the final reaction:
$\frac{\text{moles of } S_8 \text{ reacted}}{1} = \frac{\text{moles of water produced}}{16}$

Given: moles of S_8 reacting = 1

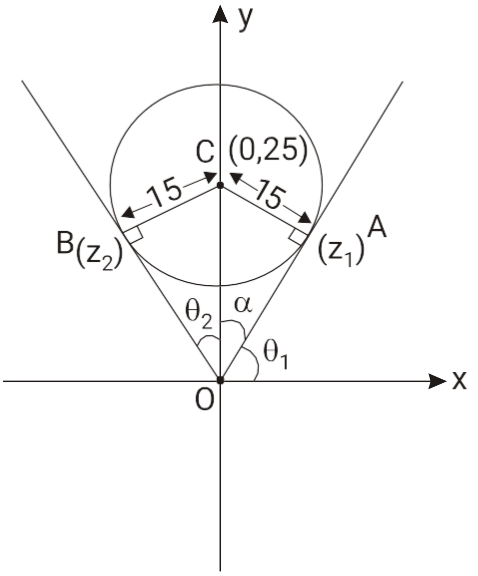
→ moles of water produced = 16

→ Mass of H_2O produced = (16 mol) × (18 gm/mol) = 288 gm

51. Given that

$|z - 25i| \le 15$ .... (i)

Which represents all the points on or inside of the circle with center $C(0, 25)$ & radius $r = 15$



From the figure, we can see that

$\max. amp(z) = amp(z_2)$ & $\min. amp(z) = amp(z_1)$

Now

$amp(z_1) = \vartheta_1$

$\rightarrow \vartheta_1 = \frac{\pi}{2} - \alpha$

$\rightarrow \cos\vartheta_1 = \cos\left(\frac{\pi}{2} - \alpha\right)$

$\rightarrow \cos\vartheta_1 = \sin(\alpha)$

$\rightarrow \cos\vartheta_1 = \frac{AC}{OC}$

$\rightarrow \cos\vartheta_1 = \frac{15}{25}$

$\rightarrow \cos\vartheta_1 = \frac{3}{5}$

$\rightarrow \vartheta_1 = \cos^{-1}\left(\frac{3}{5}\right)$

$\rightarrow amp(z_1) = \cos^{-1}\left(\frac{3}{5}\right)$

Also,

$amp(z_2) = \frac{\pi}{2} + \vartheta_2$

$\rightarrow \sin(\vartheta_2) = \frac{BC}{OC}$

$\rightarrow \sin(\vartheta_2) = \frac{15}{25}$

52. Let $P = \lim_{x \to 0} \frac{\cos^2(1 - \cos^2(1 - \cos^2(\ldots\ldots - \cos^2(x))))\ldots\ldots)}{\sin\left\{\pi\left(\frac{\sqrt{x+4} - 2}{x}\right)\right\}}$

$= \lim_{x \to 0} \frac{\cos^2 \sin^2(1 - \cos^2(\ldots\ldots \cos^2(x))\ldots\ldots}{\sin\left\{\pi\left(\frac{\sqrt{(x+4)} - 2}{x}\right)\right\}}$

$= \lim_{x \to 0} \frac{\cos^2(\sin^2(\sin^2(\ldots\ldots(\sin^2(x))\ldots\ldots}{\sin\left\{\pi\left(\frac{\sqrt{(x+4)} - 2}{x}\right)\right\}} \cdot \frac{x}{\left\{\pi\left(\frac{\sqrt{(x+4)} - 2}{x}\right)\right\}} \left\{\pi\left(\frac{\sqrt{(x+4)} - 2}{x}\right)\right\}$

$= \lim_{x \to 0} \frac{\cos^2(\sin^2(\sin^2)\ldots\ldots(\sin^2(x))\ldots\ldots)}{\frac{\sin\left\{\pi\left(\frac{\sqrt{(x+4)} - 2}{x}\right)\right\}}{\pi\left(\frac{\sqrt{(x+4)} - 2}{x}\right)}} \cdot \frac{(\sqrt{x+4} + 2)}{\pi}$

$= \frac{\cos^2 0}{1} \cdot \frac{(2+2)}{\pi} = \frac{4}{\pi}$

53. Put $x + y = t \rightarrow yt^2 = x$ or

$y = \frac{x}{t^2} \rightarrow x = \frac{t^3}{t^2+1}$; $y = \frac{t}{t^2+1}$

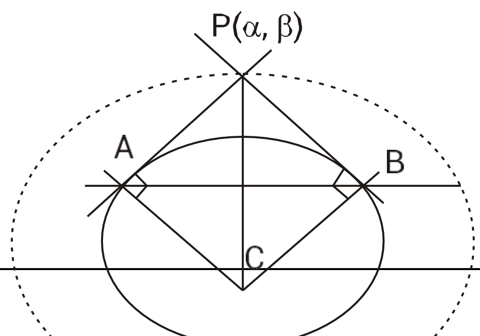
$I = \int \frac{dx}{x + 3y} = \int \frac{dx}{\frac{t^3}{t^2+1} + \frac{3t}{t^2+1}} = \int \frac{t^2+1}{t^3+3t} dx$

Also $dx = \frac{(t^2+1)3t^2 - t^3 x 2t}{(t^2+1)^2} dt = \frac{3t^2 + t^4}{(t62+1)} dt$

$I = \int \frac{t^2(t^2+3)}{(t^2+1)^2} \cdot \frac{t^2+1}{t(t^2+3)} dt$

$= \int \frac{t}{(t^2+1)} dt = \frac{1}{2}\log[t^2 + 1] + c$

54. The tangents from (α, β) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ intersect at right angle. Locus of the point of intersection of normal at the point of contact of the two tangent is



$\rightarrow \sin(\vartheta_2) = \frac{3}{5}$

$\rightarrow amp(z_2) = \frac{\pi}{2} + \sin^{-1}\left(\frac{3}{5}\right)$

Now the required value is

$|\max. amp(z) - \min. amp(z)|$

$= \frac{\pi}{2} + \sin^{-1}\frac{3}{5} - \cos^{-1}\frac{3}{5}$

$= \frac{\pi}{2} + \frac{\pi}{2} - \cos^{-1}\frac{3}{5} - \cos^{-1}\frac{3}{5}$

$= \pi - 2\cos^{-1}\frac{3}{5}$

Let the point of intersection of normal be $C(h, k)$
Clearly, AB is chord of contact to the tangents w.r.t P

(α, β)

Therefore, equation of AB is $T=0$ w.r.t. $P(\alpha,\beta)$

$$\frac{x\alpha}{a^2}+\frac{y\beta}{b^2}=1 \quad \ldots(\text{i})$$

Clearly $PACB$ is rectangle, therefore the mid-point of PC lies on AB
Now, mid-point of PC is
$M\left(\frac{\alpha+h}{2},\frac{\beta+k}{2}\right)$ which lies on (i)
Thus, put in eq.(i)

$$\frac{\left(\frac{\alpha+h}{2}\right)\alpha}{a^2}+\frac{\left(\frac{\beta+k}{2}\right)\beta}{b^2}=1 \quad \ldots(\text{ii})$$

Also AB is a chord with mid-point $M\left(\frac{\alpha+h}{2},\frac{\beta+k}{2}\right)$

Now equation of the chord bisected at M can also be given as $T=S_1$ w.r.t. $M\left(\frac{\alpha+h}{2},\frac{\beta+k}{2}\right)$ i.e.

$$\frac{\left(\frac{\alpha+h}{2}\right)\alpha}{a^2}+\frac{\left(\frac{\beta+k}{2}\right)\beta}{b^2}-1=\frac{\left(\frac{\alpha+h}{2}\right)^2}{a^2}+\frac{\left(\frac{\beta+k}{2}\right)^2}{b^2}-1 \quad \ldots(\text{iii})$$

Solving (2)and (3) we get

$$\to \frac{\beta}{\alpha}=\frac{k}{h}\to \beta x=\alpha y$$

55. Using $\frac{3\sin 76^\circ\cdot\sin 16^\circ+\cos 76^\circ\cos 16^\circ}{\cos 76^\circ\sin 16^\circ+\sin 76^\circ\cos 16^\circ}$

$$=\frac{2\sin 76^\circ\sin 16^\circ+[\sin 76^\circ\sin 16^\circ+\cos 76^\circ\cos 16^\circ]}{\sin 92^\circ}$$

$$=\frac{\cos 60^\circ-\cos 92^\circ+\cos 60^\circ}{\sin 92^\circ}$$

$$=\frac{1-\cos 92^\circ}{\sin 92^\circ}$$

$$=\frac{2\sin^2 46^\circ}{2\sin 46^\circ\cos 46^\circ}$$

$$=\tan 46^\circ=\cot 44^\circ$$

56.

$$\text{adj(adj A)}=\begin{bmatrix} a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{bmatrix}$$

$$\Rightarrow |\text{adj(adj A)}| = \begin{vmatrix} a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}^2$$

$\left[\vec{a}\cdot\vec{a}\ \ \vec{a}\cdot\vec{b}\ \ \vec{a}\cdot\vec{c}\right]$ $\left[1 \quad 1/2 \quad 1/2\right]$ 1

58. P(exaclty one of A or B)
$=P(A\cup B)-P(A\cap B)=p \quad ..(1)$
P(exaclty one of B or C)
$=P(B\cup C)-P(B\cap C)=p \quad ..(2)$
P(exaclty one of C or A)
$=P(C\cup A)-P(C\cap A)=p \quad ..(3)$
From (1), (2) & (3)

$=P(A)+P(B)-2P(A\cap B)=p \quad ..(4)$
$=P(B)+P(C)-2P(B\cap C)=p \quad ..(5)$

$=P(C)+P(A)-2P(C\cap A)=p \quad ..(6)$
Adding
(4), (5) & (6)

$$P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)=\frac{3p}{2}$$

Now,
$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)$

$$=\frac{3p}{2}+p^2=\frac{3p+2p^2}{2}$$

59. Given $f(x)=ax+b \therefore f'(x)=a$

Since $a<0, f(x)$ is a decreasing function
$\therefore f(-1)=2$ and $f(1)=0$

$\to -a+b=2$ and $a+b=0 \therefore a=-1$ and $b=1$.
Thus $f(x)=-x+1$
Clearly
$f(0)=1, f\left(\frac{1}{4}\right)=\frac{3}{4}, f(-2)=3, f\left(\frac{1}{3}\right)=\frac{2}{3}, f(-1)=2$

Also, $A=\frac{1+\cos 2\vartheta}{2}\left(\frac{1-\cos 2\vartheta}{2}\right)^2$

$$=\frac{1}{2}+\frac{1}{2}\cos 2\vartheta+\frac{1}{4}-\frac{1}{2}\cos 2\vartheta+\frac{1}{4}\cos^2 2\vartheta$$

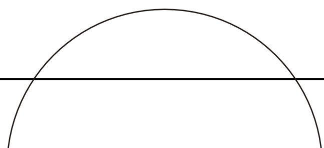
$$=\frac{3}{4}+\frac{1}{4}\left(\frac{1+\cos 4\vartheta}{2}\right)=\frac{7}{8}+\frac{1}{8}\cos 4\vartheta$$

$$\therefore \frac{3}{4}\le A\le 1\to f'\left(\frac{1}{4}\right)\le A\le f'(0)$$

60. $\sec^{-1}(\sin x)$ is real if $\sin x\le -1$ or $\sin x\ge 1$
But $-1\le x\le 1\to \sec^{-1}(\sin x)$ is real if $\sin x=-1$ or 1

$$\to x=(2n+1)\frac{\pi}{2}$$

61.



$= [\vec{a}, \vec{b}, \vec{c}]^2 = \begin{vmatrix} \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix} = \begin{vmatrix} 1/2 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{vmatrix} = \frac{}{2}$

$\Rightarrow |A|^8 = \frac{1}{2}$

57. Let $CD = a$ & $B(a, a) \rightarrow k^2 = a$

& $CG = b$ then $F = (a + b, b)$

F lies on $y = k\sqrt{x} \rightarrow b = k\sqrt{a + b}$

$\rightarrow \left(\frac{a}{b}\right)^2 + \frac{a}{b} - 1 = 0$

$\rightarrow \frac{a}{b} = \frac{\sqrt{5} - 1}{2}$

$\rightarrow \frac{b}{a} = \frac{\sqrt{5} + 1}{2}$

As $OA = OB = AB = \lambda$

Hence OAB forms an equilateral Δ

$\therefore \angle OAP = 60^\circ$

$AP = \frac{2\lambda}{}$

In Δ 5

OAB

$\cos 60^\circ = \frac{OA^2 + AP^2 - OP^2}{2OA.AP}$

$\rightarrow \frac{1}{2} = \frac{\lambda^2 + \frac{4\lambda^2}{25} - OP^2}{2\lambda\left(\frac{2\lambda}{5}\right)}$

$\rightarrow \frac{2\lambda^2}{5} = \lambda^2 + \frac{4\lambda^2}{25} - OP^2$

$\rightarrow OP^2 = \frac{19\lambda^2}{25}$

$\rightarrow OP = \frac{\sqrt{19}}{5}\lambda$

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As the distance of the variable point P from the fixed
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point O is always constant and is equal to $\frac{\sqrt{19}}{5}\lambda$

Therefore, radius of the circle described by P is $\frac{\sqrt{19}}{5}\lambda$

62. The roots of $bx^2 + cx + a = 0$ are imaginary
$\rightarrow c^2 - 4ab < 0 \rightarrow c^2 < 4ab$
Again, the coefficient of x^2 in $3b^2x^2 + 6bcx + 2c^2$ is +ve, so the minimum value of the expression

$= \frac{-(36b^2c^2 - 4(3b^2)(2c)^2)}{4(3b^2)} = \frac{12b^2c^2}{12b^2} = -c^2$

As $c^2 < 4ab$ we have $-c^2 > -4ab$.
Hence $3b^2x^2 + 6bcx + 2c^2 > -4ab\ \forall\ x \in R$

63. Given expansion is
$\rightarrow (x + a)^n = T_0 + T_1 + T_2 + T_3 + T_4 + T_5 + \ldots\ldots (i)$
&
$\rightarrow (x + a)^n = {}^nC_0(x)^n + {}^nC_1(x)^{n-1}(a) + {}^nC_2(x)^{n-2}(a)^2 + {}^nC_3(x)^{n-3}(a)^3 + {}^nC_4(x)^{n-4}(a)^4 + {}^nC_5(x)^{n-5}(a)^5 + \ldots\ldots (ii)$
Replacing a by ai in (ii), where $i = \sqrt{-1}$, we get
$\rightarrow (x + ai)^n = {}^nC_0(x)^n + {}^nC_1(x)^{n-1}(ai) + {}^nC_2(x)^{n-2}(ai)^2 + {}^nC_3(x)^{n-3}(ai)^3 + {}^nC_4(x)^{n-4}(ai)^4 + {}^nC_5(x)^{n-5}(ai)^5 + \ldots\ldots$
use $i^{4n} = 1$, $i^{4n+1} = i$, $i^{4n+2} = -1$ & $i^{4n+3} = -i$, where $n \in I$
$\rightarrow (x + ai)^n = {}^nC_0(x)^n + i.\ {}^nC_1(x)^{n-1}(a) - {}^nC_2(x)^{n-2}(a)^2 - i.\ {}^nC_3(x)^{n-3}(a)^3 + {}^nC_4(x)^{n-4}(a)^4 + i.\ {}^nC_5(x)^{n-5}(a)^5 + \ldots\ldots$
$\rightarrow (x + ai)^n = (T_0 - T_2 + T_4 - \ldots.) + i(T_1 - T_3 + T_5 - \ldots.) \ldots\ldots (iii)$
Similarly, by replacing a by $-ai$ in (ii), where $i = \sqrt{-1}$, we get
$\rightarrow (x - ai)^n = (T_0 - T_2 + T_4 - \ldots.) - i(T_1 - T_3 + T_5 - \ldots.) \ldots\ldots (iv)$
by multiplying (iii) and (iv) we get the required result
$\rightarrow (x + ai)^n(x - ai)^n = [(T_0 - T_2 + T_4 - \ldots.) + i(T_1 - T_3 + T_5 - \ldots.)][(T_0 - T_2 + T_4 - \ldots.) - i(T_1 - T_3 + T_5 - \ldots\ldots)]$
use the formula $(a + b)(a - b) = a^2 - b^2$
$\rightarrow [(x + ai)(x - ai)]^n = [(T_0 - T_2 + T_4 - \ldots.)^2 - i^2(T_1 - T_3 + T_5 - \ldots\ldots)^2]$
$\rightarrow [(x^2 - a^2i^2)]^n = [(T_0 - T_2 + T_4 - \ldots.)^2 - i^2(T_1 - T_3 + T_5 - \ldots\ldots)^2]$
put $i^2 = -1$
$\rightarrow (x^2 + a^2)^n = (T_0 - T_2 + T_4 - \ldots.)^2 + (T_1 - T_3 + T_5 - \ldots\ldots)^2$
which is the required result

64. The given line $L_1 : ax + by + c = 0$

c c

$y = \frac{\lambda}{c}\left(x + \frac{c}{a}\right) \quad ..(i)$

Equation of line QR is

$y = -\frac{c}{\lambda}\left(x + \frac{\lambda}{b}\right)$

$y = -\frac{c}{\lambda}\left(x + \frac{\lambda}{b}\right) \quad ..(ii)$

Locus of the point of intersection of (i) and (ii) is obtained by eliminating λ from (i) and (ii) Ankush swadhin
From (ii) $y + \frac{c}{b} = -\frac{c}{\lambda}x \quad ..(iii)$
Multiplying (i) and (iii) we get

$y\left(y + \frac{c}{b}\right) = -x\left(x + \frac{c}{a}\right)$
$x^2 + y^2 + \frac{c}{a}x + \frac{c}{b}y = 0$

65. Given equation $|2^{x+1} - 1| + |2^{x+1} + 1| = 2^{|x+1|},\ x \in R$
To obtain critical points equate
$2^{x+1} - 1 = 0;\ 2^{x+1} + 1 = 0;\ x + 1 = 0,\ x \in R$
Now
$2^{x+1} - 1 = 0 \Rightarrow 2^{x+1} = 1 \Rightarrow 2^{x+1} = 2^0 \Rightarrow x + 1 = 0 \Rightarrow x = -1$
And $2^{x+1} = -1$ (Not possible)
And $x = -1$
So, critical point is $x = -1$ only

Case 1: $x \le -1$

Equation becomes
$-(2^{x+1} - 1) + (2^{x+1} + 1) = 2^{-(x+1)}$
$\rightarrow 2 = 2^{-(x+1)}$
$\rightarrow x + 1 = -1$
$\rightarrow x = -2$
Case 2: If $x > -1$, so, the equation reduces to
$(2^{x+1} - 1) + 2^{x+1} + 1 = 2^{x+1}$
$\rightarrow 2^{x+1} = 0$
Not possible, hence $x = -2$

66. $\frac{dy}{dx} + y\phi'(x) = \phi(x)\phi'(x)$
It is a linear Differential equation
Hence Integrating factor: $I.F. = e^{\int \phi'(x)dx} = e^{\phi(x)}$
The solution of the differential equation is
$ye^{\phi(x)} = \int \phi(x)\phi'(x)e^{\phi(x)}dx \quad ..(i)$
Let $e^{\phi(x)} = t \Rightarrow e^{\phi(x)}\phi'(x)dx = dt$,
Also $\phi(x) = \ln t$ substitute in (i), so we get
$yt = \int \ln t dt$
$yt = t\ln t - t + c$
$y = \ln t - 1 + ct^{-1}$
$y = \phi(x) - 1 + ce^{-\phi(x)}$

$\therefore\ P = \left(-\frac{\ }{a}, 0\right), Q = \left(0, -\frac{\ }{b}\right)$
Any line L_2 is perpendicular to L_1 is
$bx - ay + \lambda = 0$

$R = \left(-\frac{\lambda}{b}, 0\right) S = \left(0, \frac{\lambda}{a}\right)$

Equation of line PS is

$y = \frac{-\frac{\lambda}{a}}{-\frac{c}{a}}\left(x + \frac{c}{a}\right)$

67. $a^2 = ar^2 \Rightarrow a = r^2$. Also $ar = 8 \Rightarrow r = 2$ and $a = 4$.
$\therefore T_6 = ar^5 = 4 \times 2^5 = 128.$

68. Equation of normal at any point

$P\left(ct, \frac{c}{t}\right)$ on $xy = c^2$ is $xt^3 - yt - ct^4 + c = 0$

If it passes through $Q(h, k)$ then

$ct^4 - ht^3 + kt - c = 0$

$\sum x_1 = \sum ct_1 = h, \sum y_1 = \sum \frac{c}{t} = k, x_1x_2x_3x_4 = -c^4$

$\frac{\sum x_1 \sum y_1}{x_1x_2x_3x_4} = -\frac{hk}{c^4}$

Here we have $h = 3, k = 2, c = 2$

$\rightarrow \frac{\sum x_1 \sum y_1}{x_1x_2x_3x_4} = -\frac{hk}{c^4} = \frac{-6}{16} = -\frac{3}{8}$

69. let $I_1 = e^{\frac{-\pi}{4}} + \int_0^{\pi/4} e^{-x} \tan^{50} x \, dx$

$I_2 = \int_0^{\pi/4} e^{-x}(\tan^{49} x + \tan^{51} x)dx$

$= \int_0^{\pi/4} e^{-x} \tan^{49} x(\sec^2 x)dx$

$= e^{-x}\frac{\tan^{50} x}{50}\Big|_0^{\pi/4} + \frac{1}{50}\int_0^{\pi/4} e^{-x} \tan^{50} x \, dx$

$= \frac{e^{-\pi/4}}{50} + \frac{1}{50}\int_0^{\pi/4} e^{-x} \tan^{50} x \, dx = \frac{I_1}{50}$ then $\frac{I_1}{I_2} = 50$

70. $f(x) = \frac{e^x \ln x 5^{(x^2+2)}(x^2 - 2)(x - 5)}{(2x - 3)(x - 4)}$

Note that at $x = 3/2$ & $x = 4$ function is not defined and in open interval $(3/2, 4)$ function is continuous.

$\lim_{x \to \frac{3}{2}^+} = \frac{(+ve)(-ve)(-ve)}{(+ve)(-ve)} \to -\infty$

$\lim_{x \to 4^-} = \frac{(+ve)(-ve)(-ve)}{(+ve)(-ve)} \to -\infty$

In the open interval $(3/2, 4)$ the function is continuous & takes up all real values from $(-\infty, \infty)$

Hence range of the function is $(-\infty, \infty)$ or R

71. $\cos^2 (45^\circ + x) + (\sin x - \cos x)^2$

$= \left[\frac{\cos x}{\sqrt{2}} - \frac{\sin x}{\sqrt{2}}\right]^2 + (\sin x - \cos x)^2$

$= \frac{3}{2}(1 - \sin 2x)$

For max, sin(2x)=-1

$= \frac{3}{2}(1 - \sin 2x) = \frac{3}{2}(1 - (-1))$

Hence, the maximum value is $= \frac{3}{2}(1 - (-1)) = 3$

The table below shows the different cases that can be taken

Value of f(c)	Value of f(a)	Number of functions
0	1	7
	2	5
	3	3
	4	2
1	0	6
	2	2
	3	1
2	0	3
		1
3	0	1
	Total Number of functions=	31

Let us explain a few cases from the table,

Case I- Subcase 1

When $f(c) = 0$ and $f(a) = 1$ ($f(a) \neq 0$ in this case as it is one-one function and $f(c)$ is already taken as 0

Now $2f(c) + 3f(a) + f(d) = f(b)$ can be expressed as

or $3(0) + 2(1) + f(d) = f(b)$

or $f(b) = 2 + f(d)$

Now as $f(b) \leq 10 \quad \{f(b) \neq 0, 1\}$

$\therefore \; 2 \leq f(d) \leq 8 \quad \{f(d) \neq 0, 1\}$

Hence total 7 possible cases in this

Case I subcase 2

Now when $f(c) = 0$ and $f(a) = 2$ ($f(a) \neq 0$ in this case as it is one-one function and $f(c)$ is already taken as 0

Now $2f(c) + 3f(a) + f(d) = f(b)$ can be expressed as

or $3(0) + 2(2) + f(d) = f(b)$

or $f(b) = 4 + f(d)$

Now as $f(b) \leq 10 \quad \{f(b) \neq 0, 2\}$

$\therefore \; 1 \leq f(d) \leq 6 \quad \{f(d) \neq 0, 1\} \quad \{f(d) \neq 2\}$

So total 5 cases

Similarly, you should take other cases from the table and check for yourself.

Using fundamental principle of counting

72. Function is defined from

73.

$f:\{a,b,c,d\} \rightarrow \{0,1,2,........,10\}$

Now the value of $f(a)$ can be 0 or 1 or 2 or $3...$ or 10
Similarly $f(b)$, $f(c)$, $f(d)$ can take any values from

0 or 1 or 2 or $3...$ or 10

We have to find the total number of one-one functions such that
$2f(a) - f(b) + 3f(c) + f(d) = 0$
or $3f(c) + 2f(a) + f(d) = f(b)$

Number of one-one functions is 31

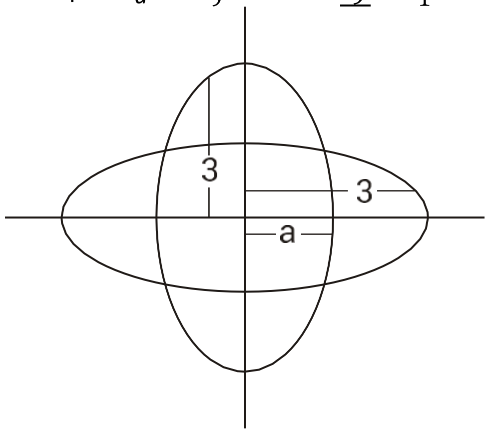
$$\lim_{x\to1} h(x) = \lim_{x\to1}\lim_{n\to\infty} \frac{x^{2x}.f(x) + x^{2m}.g(x)}{(1+x^{2n})} = g(1)$$

$$\lim_{x\to1} h(x) = \lim_{x\to1}\lim_{n\to\infty} \frac{x^{2n}.f(x) + x^{2m}.g(x)}{(1+x^{2n})} = f(1)$$

$\because \lim_{x\to1} h(x)$ exists $f(1) = g(1)$

Thus, $f(x) - g(x) = 0$ has a root at $x = 1$.

74. Given two ellipse $\frac{x^2}{a^2}+\frac{y^2}{9}=1$ and $\frac{x^2}{9}+\frac{y^2}{1}=1$



Clearly $a<3$

$\therefore\ b^2-b-3<3$

$\rightarrow (b-3)(b+2)<0$

$\therefore\ -2<b<3$

So possible integral values of $b=-1,0,1,2$

$\rightarrow$ sum = 2

75. $T_{r+1}={}^{10}C_r(2x^3)^{10-r}\left(\frac{3}{x}\right)^r$

$={}^{10}C_r 2^{10-r}3^r x^{30-4r}$

Put $r=0,1,2,.........7$ and we get $\beta=83$ Ankush swadhin