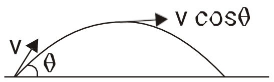
Test Id: 615107

1. $Y_n = \frac{nD\lambda}{d}$
Here, $Y_n = 10^{-3}$ m, D = 0.5 m, d = 0.5 × 10^{-3} m
$\Rightarrow \lambda = \frac{500}{n}$ nm

2. A force at any angle to velocity which is not 0° or 180° will make the object move in a plane. If the angle is either of these two, the particle will move in a straight line and may or may not return to the original point.

3. The impulse imparted to the gun is the same as the impulse imparted to the bullet, but opposite in direction.
Impulse on gun = $600 \times 10 \times 10^{-3} = 6$ Ns

4. 

At the highest point KE = $\frac{1}{2} mv^2 \cos^2 \vartheta \neq 0$

Horizontal momentum= $mv \cos \vartheta \neq 0$
Vertical momentum= 0
PE = mgh (Max at maximum height)

5. Angular momentum ($L = mvr$) of the satellite about the Sun will remain constant.
At the nearest point- P_4, distance between planet and satellite (r) will be minimum. So, at this point velocity will be maximum. Therefore kinetic energy of the plane is maximum at P_4.

6. Given that particle enters into a region having uniform electric and magnetic fields perpendicular to each other, with velocity v. Charged particle is under the influence of both magnetic force and electric force.
Force applied by the electric field = qE
Force applied by the magnetic field = qvB
It is given that accelertation of the particle is zero

$\Rightarrow F_{net} = 0$

$\Rightarrow F_B + F_E = 0$

$\Rightarrow |F_B| = |F_E|$

$\Rightarrow qvB = qE$
$\Rightarrow B = \frac{E}{v}$

7. A physical quantity $\rho = \frac{\sqrt{abc^2}}{d^3}$ is determined by measuring a, b, c and d separately with the percentage error of 2%, 3%, 2%, and 1% respectively.

Maximum percentage error in ρ is,
$\frac{\Delta\rho}{\rho} = \frac{1}{2}\frac{\Delta a}{a} + \frac{1}{2}\frac{\Delta b}{b} + \frac{\Delta c}{c} + 3\frac{\Delta d}{d}$

The percentage error contributed by c is $\frac{\Delta c}{c} = 2$

The percentage error contributed by d is, $3 \times \frac{\Delta d}{d} = 3$

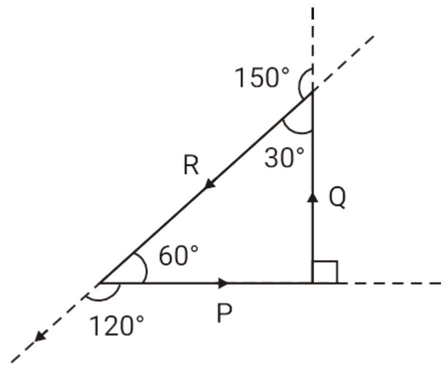
The percentage error contributed by a is minimum among a, b, c, and d.

8. In SHM, $v_{max} = a\omega$, where a = amplitude and ω = angular velocity
Averge speed of the particle in a time period is given by,
$< v > = \frac{\text{distance travelled in one oscillation}}{\text{time period}}$.
In complete oscillation, particle travells a distance of $4a$

$\Rightarrow < v > = \frac{4a}{T} = \frac{4a}{2\pi/\omega}$

$\Rightarrow < v > = \frac{2a\omega}{\pi} = \frac{2v_{max}}{\pi}$

9. Three vectors $\vec{\mathbf{P}}, \vec{\mathbf{Q}}$ and $\vec{\mathbf{R}}$ are such that $|\vec{\mathbf{P}}| = A\sqrt{2}$ and the angles between $\vec{\mathbf{P}}$ & $\vec{\mathbf{Q}}$, $\vec{\mathbf{Q}}$ & $\vec{\mathbf{R}}$, $\vec{\mathbf{R}}$ & $\vec{\mathbf{P}}$ are 90°, 150° and 120° respectively. Based on the information, 3 vectors can be represented as shown in the diagram.



i.e 3 vectors are forming a right angled triangle.

Tan 60º = $\frac{|\vec{\mathbf{Q}}|}{|\vec{\mathbf{P}}|}$

$\Rightarrow |\vec{\mathbf{Q}}| = \sqrt{3}|\vec{\mathbf{P}}|$

$\Rightarrow |\vec{\mathbf{Q}}| = \sqrt{6}A$

10. $u_{rel} = u_1 - u_2 = 0 - (-5) = 5$ ms^{-1}
t = 3 s
$a_{rel} = a_1 - a_2 = -g - (-g) = 0$ ms^{-2}

ρ max 2 a 2 b c d

The percentage error contributed by a is,

$\frac{1}{2}\frac{\Delta a}{a} = \frac{1}{2} \times 2 = 1.$

The percentage error contributed by b is

$\frac{1}{2}\frac{\Delta b}{b} = \frac{1}{2} \times 3 = \frac{3}{2}.$

$s_{rel} = u_{rel}t + \frac{1}{2} a_{rel}t^2$

$\Rightarrow s_{rel} = 5 \times 3 = 15$ m ($\because a_{rel} = 0$)

So, $\boxed{s_{rel} = 15 \text{ m}}$

11. $Y = \overline{(A+B)}.\overline{(AB)}$

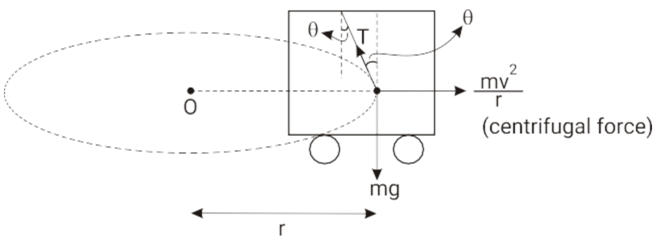
$= \overline{(AB)}.(AB)$

Let $AB = X$

$Y = \overline{X}.X = 0$

12. Given that a car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 ms^{-1}. A plumb bob is suspended from the roof of the car by a string of length 1 m. We have to find the angle made by the string with vertical ($g = 10$ ms^{-2}).

Let us use the frame of reference of the car (a rotating frame of reference). A centrifugal force acts away from the centre of the circle in this frame.
We have taken the radius of the circle (in which the plumb bob moves) to be r.



The bob is at rest in this frame. So, the net force on it is zero.
So, we get, $T\cos\vartheta = mg$.......... (i)

And, $T\sin\vartheta = \frac{mv^2}{r}$ .... (ii)

Dividing (ii) by (i), we get, $\tan\vartheta = \frac{v^2}{rg}$.

$\Rightarrow \tan\vartheta = \frac{100}{10(10)} = 1$

$\Rightarrow \vartheta = 45°$

13. According to Lenz's Law, current is induced in the counterclockwise direction and be increasing to oppose the change in the decreasing flux.

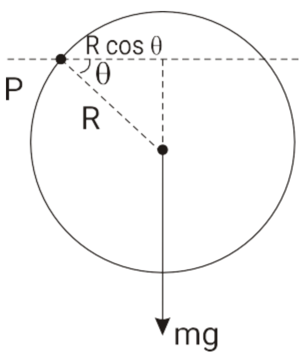
14. $f^{-1} = v^{-1} - u^{-1}$

$-f^{-2}df = -v^{-2}dv - u^{-2}du$

$\frac{df}{f^2} = \frac{dv}{v^2} + \frac{du}{u^2}$

$df = f^2\left[\frac{dv}{v^2} + \frac{du}{u^2}\right]$

15.



Uniform disc is undergoing pure rotation about the hinge P. At the instant when disc is rotated through an angle ϑ, net torque acting on the disc about the axis theorem)

According to torque equation, $\tau = I\alpha$

$\Rightarrow mgR\cos\vartheta = \frac{3}{2}mR^2\alpha$

$\Rightarrow \alpha = \frac{2g\cos\vartheta}{3R}$

16. $T' = T + \frac{96}{100}T = 1.96\ T$

$f \propto \sqrt{T}$

$\frac{f+10}{f} = \sqrt{\frac{1.96T}{T}}$

$f + 10 = 1.4\ f$

$f = 25$ Hz

Now, $f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

$\frac{f_1}{f_2} = \frac{l_2}{l_1} = \frac{1 + 0.25l}{l}$

$f_2 = \frac{25}{1.25} = 20$ Hz

Change in frequency = 25 − 20 = 5 Hz

17. Given that the energy spectrum of a black body exhibits a maximum around a wavelength λ_0. According to Wein's displacement law, product of wavelength corresponding to maximum intensity and temperature of the black body is constant. $\lambda_m T = 0.282$ cm k

Given that the temperature of the black body is now changed such that the energy is maximum around a wavelength $\frac{3\lambda_0}{4}$. Let the initial temperature is T_1 and final temperature is T_2.

According to Wein's displacement law, $\lambda_0 T_1 = \frac{3\lambda_0}{4} T_2$

$\Rightarrow \frac{T_1}{T_2} = \frac{3}{4}$

According to Stefan-Boltzman law, power emitted by black body, $u \propto T^4$

$\therefore \frac{u_1}{u_2} = \frac{T_1^4}{T_2^4} = \frac{3^4}{4^4} = 81 : 256$

$\Rightarrow u_2 = \frac{256}{81}u_1$

18. Given, initial wavelength used = λ and initial stopping potential = V.

Final wavelength used = 2λ, final stopping potential = $\frac{V}{4}$

.

Consider, work function of the metallic surface = ϕ.

Threshold wavelength = λ_0. Relation between threshold wavelength and work function is $\phi = \frac{hc}{\lambda_0}$.

hinge P is $mgR\cos\vartheta$.

Moment of inertia of disc about an axis passing through hinge P, $I_P = \frac{mR^2}{2} + mR^2 = \frac{3mR^2}{2}$. (using parallel

From Einstein's photoelectric equation we can write

Intial case, $eV = \frac{hc}{\lambda} - \phi$.......... (i)

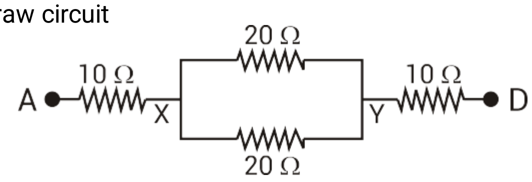
Final case, $\frac{eV}{4} = \frac{hc}{2\lambda} - \phi$ .... (ii)

(i) $-4\times$ (ii): $0 = -\frac{hc}{\lambda} + 3\phi$

$\Rightarrow 3\left(\frac{hc}{\lambda_0}\right) = \frac{hc}{\lambda}$

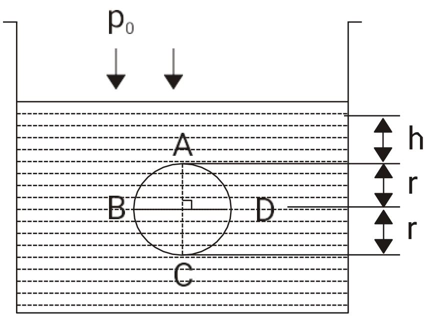
$\Rightarrow \lambda_0 = 3\lambda.$

19. Redraw circuit



20. Given,

Points B and D are in same horizontal line and points A and C are in same vertical line.



Points at same height have same pressure, points with height difference say 'h' will have difference of ρgh.

$\Rightarrow p_B = p_D$

Let radius of circle is r and point A is at a depth of h from the free surface.

$\Rightarrow p_A = P_0 + h\rho g$

$\Rightarrow p_B = p_D = P_0 + (h + r)\rho g$

$\Rightarrow p_C = P_0 + (h + 2r)\rho g$

Then, $p_C + p_A = [P_0 + (h + 2r)\rho g] + [P_0 + h\rho g]$

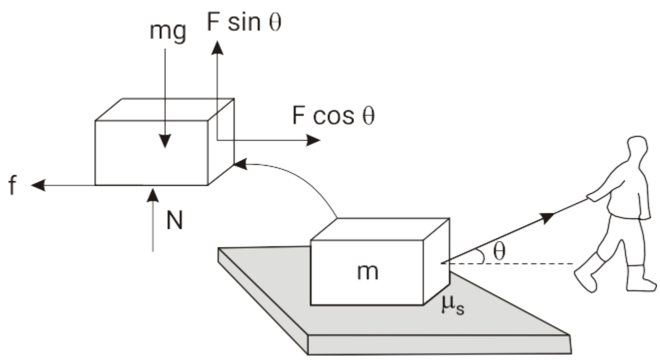
$\Rightarrow p_C + p_A = P_0 + h\rho g + 2r\rho g + P_0 + h\rho g$

$\Rightarrow p_C + p_A = 2[P_0 + (h + r)\rho g]$

$\Rightarrow \frac{p_C + p_A}{2} = P_0 + (h + r)\rho g$

$\therefore \frac{p_C + p_A}{2} = p_B = p_D.$

21.



• It tends to move the block along the surface.

The minimum value of F occurs at an angle ϑ at which the normal reaction is reduced such that the horizontal component of the applied force becomes just equal to the limiting friction.

i.e $F\cos\vartheta = \mu_s N$

$\Rightarrow F\cos\vartheta = \mu_s(mg - F\sin\vartheta)$

$\Rightarrow F(\cos\vartheta + \mu_s\sin\vartheta) = \mu_s mg$

$\Rightarrow F = \frac{\mu_s mg}{(\cos\vartheta + \mu_s\sin\vartheta)}$ .... (i)

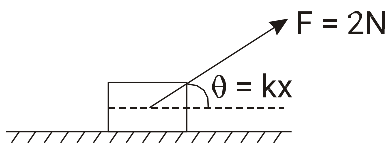
For the minimum value of F, $(\cos\vartheta + \mu_s\sin\vartheta)$ should be maximum.

Maximum value of $(\cos\vartheta + \mu_s\sin\vartheta)$ is $\sqrt{1+\mu^2_s}$

$\Rightarrow F_{min} = \frac{\mu mg}{\sqrt{1+\mu^2}}$

$\Rightarrow F_{min} = \frac{\frac{1}{\sqrt{3}} \times 10}{\frac{2}{\sqrt{3}}} = 5$ N.

22. The situation given in the question is represented in this picture.



Only the horizontal component of the force is doing work.

So, we get,

$\int F\cos\vartheta dx = \frac{1}{2}mv^2 = E$

$\Rightarrow E = \int_0^x 2\cos(kx)dx$

$\Rightarrow E = \frac{2}{k}[\sin kx]_0^x$

$\Rightarrow E = \frac{2}{k}\sin kx$

$\therefore E = \frac{2\sin\vartheta}{k}.$

23. $f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$

$\Rightarrow f \propto \frac{1}{\sqrt{m}}$

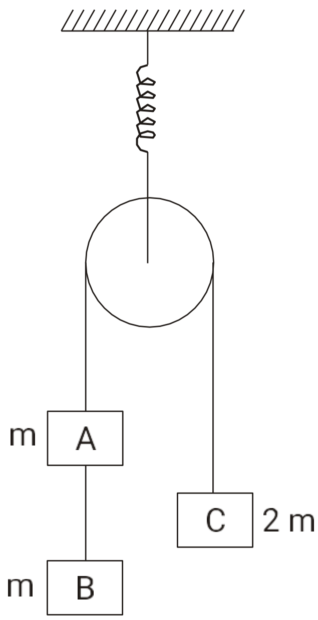
$\Rightarrow \frac{f_1}{f_2} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{9m}{m}} = 3$

We wish to calculate the value of ϑ at which minimum force is required to move the block.

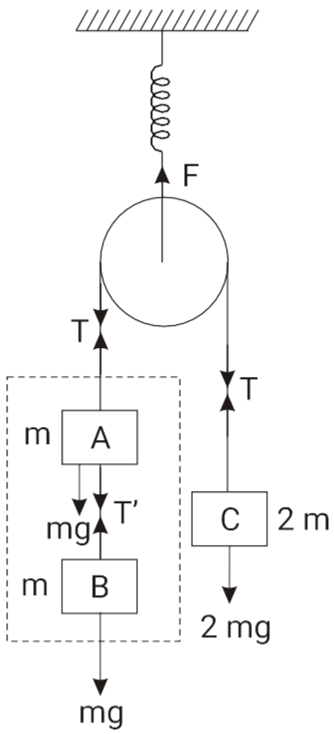
From the free body diagram of the block, we can easily understand that the applied force has two effects:

• It reduces the normal reaction thus, reduces the frictional force.

24. In the figure shown, the pulley and the spring are ideal and the strings are light and inextensible. Initially all the bodies are at rest when the string connecting A and B is cut. We have to find the initial acceleration (in m/s^2) of the pulley.

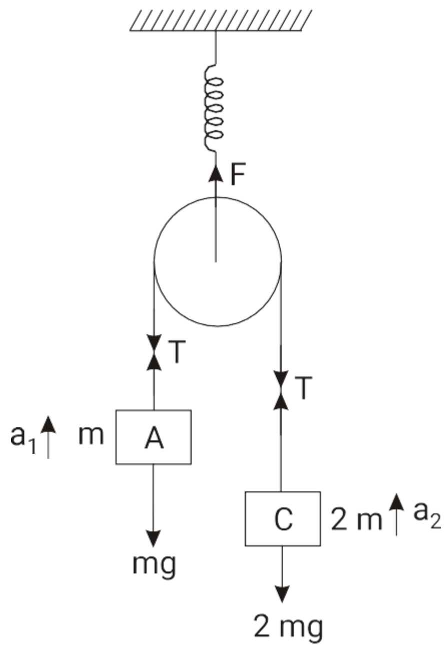


The initial situation is shown below.



The masses on both the sides of the pulley are the same. So, the system is in equilibrium. From the equilibrium of C, $T = 2mg$.
The pulley is at rest. So, we get, $F = 2T$, where F is the spring force.
$\Rightarrow F = 4mg$

The situation just after cutting the string is shown below.

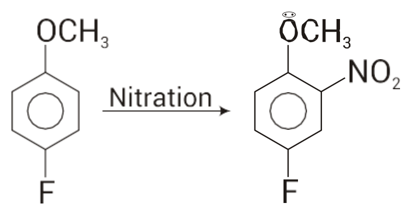


The spring force does not change instantly. Just after cutting the string, the spring force will be $4mg$. Since the pulley is massless, the net force on it is zero even when it is moving. So, the tension tension in the string passing over the pulley is still $2mg$.
From constraint relations, the acceleration of the pulley is, $a_{pulley} = \dfrac{a_1 + a_2}{2}$.
Now,for the block A, $T - mg = ma_1$.
$\Rightarrow 2mg - mg = ma_1$
$\Rightarrow a_1 = g$
Similarly, for the block C, $T - 2mg = 2ma_2$.
$\Rightarrow 2mg - 2mg = 2ma_2$
$\Rightarrow a_2 = 0$
So, the acceleration of the pulley is,
$a_{pulley} = \dfrac{a_1 + a_2}{2} = \dfrac{g + 0}{2}$.

$\Rightarrow a_{pulley} = 5\ m/s^2$

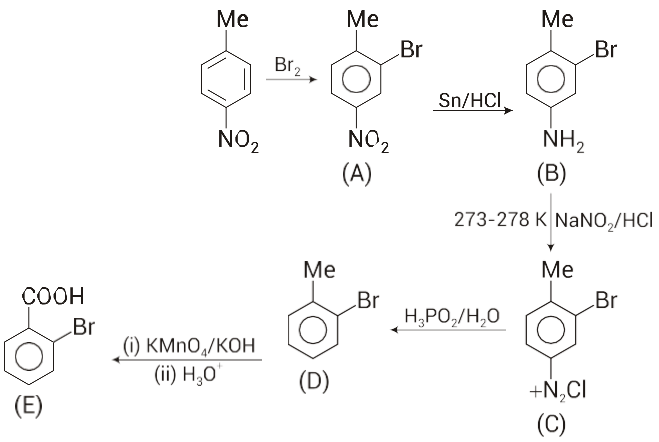
25. $\eta V_P i_P = V_s i_s$
$2300 \times 5 \times 0.9 = 230 \times i_0$
$\Rightarrow i_0 = 45$ A

26. This is SEAr reaction :



$-\ddot{O}CH_3$ has + M effect. Hence activating group at ortho and para position where as —F is − I > + M hence deactivating and meta directing group.
If activating and deactivating groups are there on benzene then position of substitution is governed by activating group hence substitution is at ortho of —OCH_3.

27.



28. The balanced reaction of Zn and HCl is given below:
$Zn + 2HCl \rightarrow ZnCl_2 + H_2(g)$

We know, 1 mole of any gas produces 22400 mL at STP.

⇒ Number of moles of 1.12 mL of H_2 gas at STP = $\frac{1.12}{22400}$

From the balanced reaction, we can say,
1 mole of H_2 gas is produced by 1 mole of Zn

⇒ $\frac{1.12}{22400}$ mole of H_2 gas is produced by $\frac{1.12}{22400}$ mole of Zn.

⇒ Mass of Zn required = $\frac{1.12}{22400} \times 65$ g

⇒ Mass of Zn required = 32.5×10^{-4} g

⇒ Option (C) is *CORRECT.*

29. Elevation in boiling point, $\Delta T_b = i \times K_b \times m$
Molality of NaCl solution

$$= \frac{n}{w} \times 1000 = \frac{\frac{58.5}{58.5}}{w_{H_2O}} \times 1000 = \frac{1000}{w_{H_2O}}$$

Molality of $C_6H_{12}O_6$ solution = $\frac{\frac{180}{180} \times 1000}{w_{H_2O}} = \frac{1000}{w_{H_2O}}$

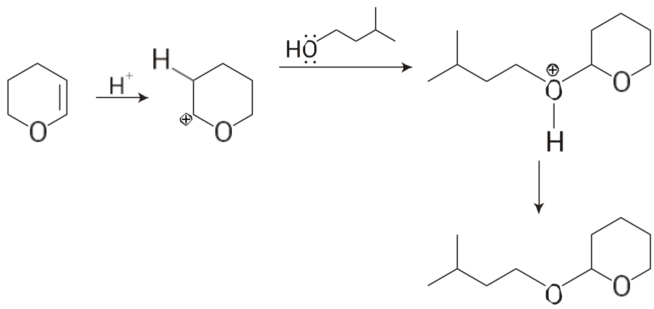
Both solutions have the same molality but values of i, but Vant Hoff factor for NaCl and glucose are 2 and 1 respectively.
Hence, NaCl will show higher elevation in boiling point and the statement (a) is correct.

30. Chlorination of CH_4 takes place either in the presence of light or at high temperature, and not in the dark because, in darkness, Cl free radicals are not produced.

31. Nessler's Reagent Reaction:

Therefore, under a reduced pressure of 12mm Hg, glycerol can be distilled at 453K without decomposition.

33.



34. (A) On strong heating, Ag_2CO_3 does not give Ag_2O as silver oxide can further decompose forming silver as shown in the reaction below.

$2Ag_2CO_3 \xrightarrow{\Delta} 4Ag + 2CO_2 + O_2$

⇒ The given statement is incorrect.

(B) Thermal decomposition of ammonium dichromate yields N_2. In fact, thermal decomposition of an ammonium salt with a highly oxidizing anion tends to produce N_2 or sometimes an oxide of nitrogen.
However, the thermal decomposition of ammonium chloride will form NH_3 and HCl. The reactions taking place are:

$(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + Cr_2O_3 + H_2O$

$NH_4Cl \xrightarrow{\Delta} NH_3 + HCl$

⇒ The given statement is incorrect.

(C) Be exists as Be^{2+} in its halides. Due to the small size and high charge density Be^{2+} has high polarizing power as a result of which the halides of Be with larger halogens like Cl, Br, and I have significant covalent character in accordance with Fajan's Rule.

(D) Among the 3d series elements Zinc has the lowest melting point. In fact, the elements in the same group as Zn (i.e. Cd, Hg) have the lowest melting points in their respective transition series. This is so because these elements have completely filled penultimate d orbitals and also 2 electrons in the valence s-orbital. Thus, there is a considerably weaker tendency to share the valence electrons and form strong metallic bonds. Due to the weaker metallic bonds, these elements have lower than expected melting points.

35. $K_P = 0.667 \text{ atm} = \frac{2}{3} \text{ atm} = \frac{4\alpha^2}{1-\alpha^2} \cdot P = \frac{4\alpha^2}{1-\alpha^2} \cdot \frac{1}{2}$

So, $\frac{4\alpha^2}{1-\alpha^2} = \frac{4}{3}$

⇒ $3\alpha^2 = 1 - \alpha^2$

$\underset{\text{Nessler's Reagent}}{2K_2HgI_4} + NH_3 + 3KOH \rightarrow \underset{\text{(Iodine of Million's base /Brown precipitate)}}{HgO \cdot Hg(NH_2)I} + 7KI + 2H_2O$

32.

Distillation under reduced pressure is used to purify liquids having very high boiling points and those, which decompose at or below their boiling point.
Glycerol, at normal pressure, the b.p. of glycerol is 563K but it decomposes at this temperature.

$\Rightarrow \alpha^2 = \frac{1}{4}$

$\Rightarrow \quad \alpha = \frac{1}{2}$

36. All forms of Vitamin B are important for making sure that the body's cells are functioning properly.
They help the body convert food into energy (metabolism), create new blood cells, and maintain healthy skin cells, brain cells, and other body tissues.

37. The molar mass of Ag = 108 g.mol^{-1}
The equivalent weight of Ag =
$\frac{\text{Molar mass of Ag}}{\text{n-factor}} = \frac{108}{1} = 108$
According to Faraday's first law of electrolysis,
$\frac{\text{mass of substance discharged}}{\text{E.W. of the substance}} = \frac{i \times t}{96500}$
Given:
mass of Ag deposited = 0.746 g

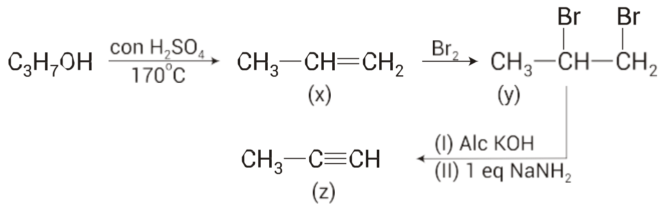
& time for which current is passed = 548 s
Putting the values we get:

$$0.746 = \frac{108}{96500} \times i \times 548$$

⇒ Current (i) = 1.22 A

⇒ Option (A) is *CORRECT.*

38. Step 1:
Dehydration of alcohol by acid to form alkene
Step 2:
Addition of bromine to alkene to form vicinal di bromide
Step 3:
Dehydrohalogenation of vicinal dibromide to form alkyne



39. Statement - I ⇒ Correct
Statement - II ⇒ False
Ga is used to measure high temperature

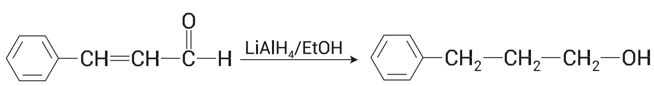
40.
$$h\nu = \Delta E = 13.6 \times Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

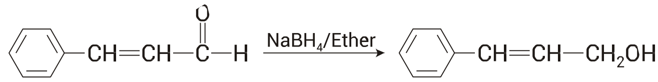
⇒ $\nu \propto Z^2$

⇒ $(\nu)^{1/2} \propto Z \left(n = \frac{1}{2}\right)$

41. HI/Red P_4 reduces the aldehyde into an alkane, but it doesn't affect the double bond.

$LiAlH_4$ / EtOH also reduces the double bond along with the aldehyde group when the double bond is in resonance with the phenyl ring as well as the aldehyde group





42. Mendeleev arranged elements in horizontal rows and vertical columns in order of their increasing atomic properties occupied by the same vertical column or

group. He ignored the order of atomic weight thinking that the atomic measurements might be incorrect and placed the elements with similar properties.

Thus, both assertion and reason are correct but reason is not the correct explanation of assertion.

43. Ligands are such a group of atoms (charged or uncharged) which can donate one or more lone pair of

electrons to another atom / ion.
Since ligands can donate lone pair of electrons, therefore all ligands are lewis base.
Remember:
Electron pair donors are Lewis base and electron pair acceptors are Lewis Acid.

44. All are having same equivalents

$$\underbrace{2NaOH}_{2} = \underbrace{H_2SO_4}_{1 \times 2} = \underbrace{Na_2SO_4}_{1 \times 2}$$
$$\text{n-factor} = 1 \qquad 2 \qquad 2$$

Both Statement I and Statement II are correct Statement II is correct explanation of Statement I.

45. $\dot{\underset{\cdot}{N}} = \ddot{\underset{\cdot\cdot}{O}}$

For NO, the octet rule is not followed due to the presence of odd electrons on N.

46. The enthalpy change for the given reaction,ΔH = −57.8 kJ mol^{-1}
Entropy change for the reaction,ΔS = −176.0 JK^{-1} =
$-\frac{176}{1000}$ KJ K^{-1}
Using Gibb's free energy equation,
$\Delta G = \Delta H - T\Delta S$
⇒ $\Delta G = -57.8 - \frac{298(-176)}{1000}$
⇒ $\Delta G = -57.8 + 52.448$
$NaBH_4$/ Ether reduces the aldehyde into primary alcohol, but it doesn't affect the double bond.

$\Rightarrow \Delta G = -5.352 KJ\ mol^{-1}$

$\Rightarrow |\Delta G| = 5.352$ KJ/mol

$\Rightarrow$ Answer = 5 (after rounded off to the nearest integer)

47. H_2S in acidic medium behaves as the precipitating agent for Group 2 cations.

The cations which belong to group 2 are:
Hg^{2+}, Pb^{2+}, Bi^{3+}, Cu^{2+}, Cd^{2+}, As^{3+}, Sb^{3+}, Sn^{2+}.

Thus, among the given cations, the following will precipitate as respective sulphides on passing H_2S in an acidic medium.
Pb^{2+}, Cu^{2+}, Hg^{2+}, Cd^{2+}.

Now, among the 4 sulphides precipitated among the given cations, CdS is yellow in colour while the other 3 sulphide are black in colour.

$\Rightarrow$ Answer = 3.

48. $C_8H_{18}O_2(g) \longrightarrow 2CH_3COCH_3(g) + C_2H_6(g)$

t = 0	800	0	0
t = t	800–P	2P	P

Given : $P_{acetone} = 1400$

$\Rightarrow 2P = 1400 \Rightarrow P = 700$

Given : $t_{1/2} = 80$ min $\Rightarrow k = \frac{\ln 2}{80}$

$\therefore \frac{\ln 2}{80} \times t = \ln\left(\frac{800}{100}\right)$

$\Rightarrow \frac{\ln 2}{80} \times t = \ln 8 = 3 \ln 2$

$\Rightarrow t = 80 \times 3 = 240$ min

49. $5e^- + 8H^+ + \underset{10^{-1}}{MnO_4^-} \longrightarrow \underset{10^{-3}}{Mn^{2+}} + 4H_2O$

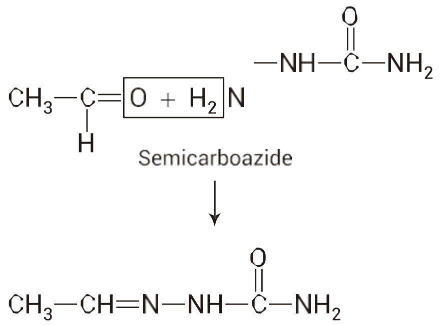
$1.282 = 1.54 - \frac{0.059}{5}\log \frac{10^{-3}}{10^{-1}(H^+)^8}$

$-0.258 = \frac{-0.059}{5}(-2 + 8pH)$

21.8644 = (-2 + 8 pH)
23.8644 = 8 pH

pH = 2.98 ≈ 3

50.



51. Let P(computer turns out to be defective given that it is produced in plant T_2) = x,
$\Rightarrow P$(computer turns out to be defective given that it is produced in plant T_1) = $10x$

Given: $\frac{7}{100} = \frac{1}{5}(10x) + \frac{4}{5}x$ (By Total Probability Theorem)

$\Rightarrow 7 = 200x + 80x \Rightarrow x = \frac{7}{280}$

Let us define the following events:
A: Computer is not defective
B: Computer is produced in T_1
C: Computer is produced in T_2

Now, required probability

$= P(C/A) = \frac{P(C \cap A)}{{}^4(1-x)} = \frac{P(A/C).P(C)}{P\quad(A)}$

$^4\left(\frac{273}{}\right)$ $\quad P(A/B).P(B) + P(A/C).P(C)$

$\therefore \alpha - \beta = \frac{\pi}{2} \quad ..(i)$

also $\alpha x^2 + \beta x + \frac{\pi}{2} = 0$ & $x = 3$ only

$\therefore 9\alpha + 3\beta = -\frac{\pi}{2} \quad ..(ii)$

$12\alpha = \pi \Rightarrow \alpha = \frac{\pi}{12}$

53. We have
The line: $y = mx + c$
And the parabola: $y^2 = 4a(x + a)$

Solve both the curves simultaneously

i.e. $y^2 = 4a(x + a)$
or $(mx + c)^2 = 4a(x + a)$
or $m^2x^2 + c^2 + 2mxc = 4ax + 4a^2$
or $m^2x^2 + (2mc - 4a)x + c^2 - 4a^2 = 0$

As line is tangent to the parabola, its $D = 0$

$\Rightarrow (2mc - 4a)^2 = 4.m^2(c^2 - 4a^2)$
$\Rightarrow m^2c^2 + 4a^2 - 4mca = m^2c^2 - 4m^2a^2$
$\Rightarrow a - mc = -m^2a$
$\Rightarrow c = \frac{a}{m} + ma$

54. Given :$f(x) = \log_{\sqrt{x}}\left(\frac{1}{x}\right)$

Use the property $\log_b a = \frac{\log a}{\log b}$

$= \frac{\log\left(\frac{1}{x}\right)}{\log\sqrt{x}} = \frac{(-1)\log x}{\frac{1}{2}\log x} = -2$

$\Rightarrow f'(x) = 0.$

55. See the figure since, $OP \perp PY$

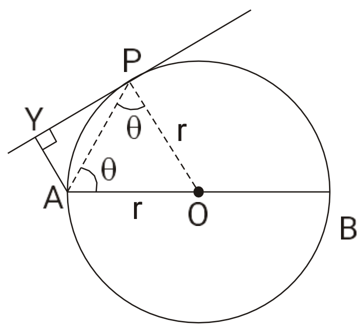
$\therefore \angle APY = 90^\circ - \vartheta$

where $OPA = \vartheta$

$\therefore \angle PAY = \vartheta$

Now in
$\Delta OPA, AP^2 = r^2 + r^2 - 2r.r\cos(\pi - 2\vartheta) = 4r^2\cos^2\vartheta$

$\therefore AP = 2r\cos\vartheta \Rightarrow PY = AP\sin\vartheta = r\sin 2\vartheta$ and
$AY = AP\cos\vartheta = 2r\cos^2\vartheta$



$\therefore$ Area of ΔAPY, $\Delta = \frac{1}{2} PY \cdot AY = r^2 \sin 2\vartheta \cos^2 \vartheta$

$$= \frac{\frac{1}{5}}{\frac{1}{5}(1-10x)+\frac{4}{5}(1-x)} = \frac{\frac{1}{5}\,280}{\frac{1}{5}\left(\frac{280-70}{280}\right)+\frac{4}{5}\left(\frac{273}{280}\right)} = \frac{4\times 273}{210+4\times 273} = \frac{2\times 273}{105+2\times 273} = \frac{546}{651} = \frac{78}{93}$$

52. $\sin^{-1}(\sin\vartheta) > \cos^{-1}(\sin\vartheta)$

For $Q \rightarrow \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

$$\frac{d\Delta}{d\vartheta} = r^2[2\cos 2\vartheta \cos^2\vartheta - \sin^2 2\vartheta] = 0 \Rightarrow \vartheta = \frac{\pi}{2}, \frac{\pi}{6}$$

$\because \vartheta \neq \frac{\pi}{2}$. Also Δ is maximum at $\vartheta = \frac{\pi}{6}$ (Check)

$\therefore \Delta_{\max} = r^2 . \frac{\sqrt{3}}{2}\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3\sqrt{3}r^2}{8}$

56. We have, $f(x) = g(x+1) \Rightarrow$

$\sum_{r=0}^{2n} a_r x^r = \sum_{r=0}^{n-1} b_r (x+1)^r + \sum_{r=n}^{2n} (x+1)^r$

$\Rightarrow \sum_{r=0}^{2n} a_r x^r = \sum_{r=0}^{n-1} b_r (x+1)^r + (x+1)^n + (x+1)^{n+1} + \ldots\ldots + (x+1)^{2n}$

$\Rightarrow \sum_{r=0}^{2n} a_r x^r = \sum_{r=0}^{n-1} b_r (x+1)^r + (x+1)^n \left\{\frac{(x+1)^{n+1}-1}{x+1-1}\right\}$

$\Rightarrow \sum_{r=0}^{2n} a_r x^r = \sum_{r=0}^{n-1} b_r (x+1)^r + \frac{(x+1)^{2n+1}-(x+1)^n}{x}$

Now, a_n = Coefficient of x^n in $\sum_{r=0}^{2n} a_r x^r$

= Coefficient of x^n in

$\left\{\sum_{r=0}^{n-1} b_r (x+1)^r + \frac{(x+1)^{2n+1}-(x+1)^n}{x}\right\}$

= Coefficient of x^n in $\left\{\frac{(x+1)^{2n+1}-(x+1)^n}{x}\right\}$ =

Coefficient is x^{n+1} in $\{(x+1)^{2n+1}-(x+1)^n\}$

= Coefficient is x^{n+1} in $(x+1)^{2n+1} = {}^{2n+1}C_{n+1}$

57. $\int \frac{\operatorname{cosec}^2 x - 2009}{\cos^{2009} x} dx$

$= \int \operatorname{cosec}^2 x \cdot (\cos x)^{2009} dx - 2009$

$\int \frac{1}{(\cos x)^{2009}} dx$

$= I_1 - I_2$

Applying by parts on I_1,

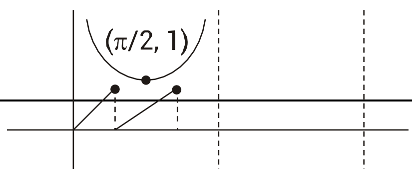
We get $\int \frac{\operatorname{cosec}^2 x - 2009}{\cos^{2009} x} dx$

$= -\frac{\cot x}{(\cos x)^{2009}} + c$

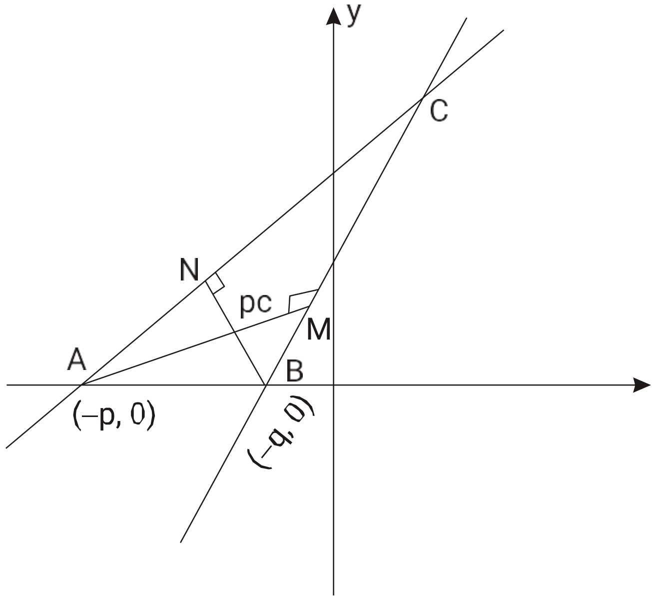
$\therefore$ $A(x) = \cot x$ and $B(x) = \cos x$

$\Rightarrow \frac{A(x)}{B(x)} = \operatorname{cosec} x = \{x\}$ for $x \in [0, 2\pi]$ then equation

has no solution as clear from the graph below



58.



We have

$(1+p)x - py + p(1+p) = 0$

$\Rightarrow \frac{x}{-p} + \frac{y}{1+p} = 1$

Also,

$(1+q)x - qy + q(1+q) = 0$

$\Rightarrow \frac{x}{-q} + \frac{y}{1+q} = 1$

Equatio of AM is

$y = -\frac{p}{1+p}(x+a)$

and equation of BM is $y = -\frac{q}{1+a}(x+p)$

$\Rightarrow \frac{p}{1+p}(x+a) = \frac{q}{1+q}(x+p)$

$\Rightarrow x = pq \quad ..(i)$

Also, $y = -\frac{p}{1+p}(pq+q)$

$\Rightarrow y = -pq \quad ..(ii)$

from (i) and (ii)

$x = -y$

$\Rightarrow x + y = 0$

Which is the requied locus representing a straight line.

59. $\sin^{-1}\left(\frac{1+x^3}{2x^{3/2}}\right)$ will exist only when $\frac{1+x^3}{2x^{3/2}} \leq 1$

As solving we get $x^{3/2} = 1 \Rightarrow x = 1$

But at $x = 1$; $\log_{3\{x\}+1}(x^2+1)$ will not exist

1 2

$\therefore$ domain $x \in \phi$

60.

$$f(x) = \begin{cases} \dfrac{\tan^2\{x\}}{(x^2 - [x]^2)}, & x > 0 \\ 1, & = 0 \\ \sqrt{\{x\}\cot\{x\}}, & x < 0 \end{cases}$$

Option-A

$$\lim_{x\to 0} f(x) = \lim_{h\to 0} \frac{\tan^2\{0+h\}}{(0+h)^2 - [0+h]^2} = \lim_{h\to 0} \frac{\tan^2 h}{h^2} = 1$$

Option-B

$$\lim_{x\to 0} f(x) = \lim_{h\to 0} \sqrt{\{0-h\}\cot\{0-h\}}$$

$= \lim_{h\to 0} \sqrt{(1-h)\cot(1-h)} = \sqrt{\cot 1}$

Option C

$\tan^{-1}(\lim_{x\to 0} f(x)) = \tan^{-1} 1 = \frac{\pi}{4}$

61. Using G.M. ≤ A.M. we get

$(a^a b^b c^c)^{\frac{1}{a+b+c}} \le \frac{a^2+b^2+c^2}{a+b+c}$, $(a^b b^c c^a)^{\frac{1}{a+b+c}} \le \frac{ab+bc+ca}{a+b+c}$

& $(a^c b^a c^b)^{\frac{1}{a+b+c}} \le \frac{ac+ba+cb}{a+b+c}$

Hence

$a^a b^b c^c + a^b b^c c^a + a^c b^a c^b \le a^2+b^2+c^2+2ab+2bc+3ca = (a+b+c)^2 = 1.$

62.

$2\cos 2\vartheta = 1$

$\cos 2\vartheta = \frac{1}{2} = \cos 60^\circ$

$2\vartheta = 60^\circ$

$\vartheta = 30^\circ$

63.

Given: $f(k) = \int_0^4 |4x - x^2 - k|\, dx$

Consider $y = 4x - x^2 - k$

Assume that one of the root of the above quadratic is $x = \alpha$ (Also shown in the figure)

Thus we have $4\alpha - \alpha^2 - k = 0$

$\Rightarrow k = 4\alpha - \alpha^2 = \alpha(4-\alpha) \quad ..(1)$

We have to find k for which the area enclosed is minimum , So apply the following steps :-

Step.1 : First draw the curve $y = 4x - x^2 - k$

Clearly $y = 4x - x^2 - k$ is a downword parabola (As coeffecient of x^2 is negative)

Also the x coordinate of the vertex for the above parabola is $x = -\frac{b}{2a} = 2$

Step 2: Now draw the curve $y = |4x - x^2 - k|$

For this, Reflect the portion of the parabola lying below x axis about the line $y = 0$ (i.e x axis) as shown below in the figure.

Step 3: we have to find the area bounded by the curve , x axis and between the lines $x = 0$ and $x = 4$

Hence the required area will be the area shaded in the figure

$\Rightarrow$ R.A $= A_1 + A_2 + A_3 + A_4$

Using symmetry we know that $A_1 = A_4$ and $A_2 = A_3$

$\therefore$ R.A $= 2(A_1 + A_2)$

Now $A_1 = \int_0^\alpha |4x - x^2 - k|\, dx$

$\Rightarrow A_1 = \int_0^\alpha -(4x - x^2 - k)\, dx$

$= -\left(\frac{4x^2}{2} - \frac{x^3}{3} - kx\right)_0^\alpha$

$= \left[-2x^2 + \frac{x^3}{3} + kx\right]_0^\alpha$

$= -2\alpha^2 + \frac{\alpha^3}{3} + k\alpha \quad ..(\text{i})$

And $A_2 = \int_\alpha^2 |4x - x^2 - k|\, dx$

$\Rightarrow A_2 = \int_\alpha^2 (4x - x^2 - k)\, dx$

$= \left(\frac{4x^2}{2} - \frac{x^3}{3} - kx\right)_\alpha^2$

$= \left[2x^2 - \frac{x^3}{3} - kx\right]_\alpha^2$

$= \left(2(2)^2 - \frac{2^3}{3} - k(2)\right) - \left(2(\alpha)^2 - \frac{\alpha^3}{3} - k(\alpha)\right)$

$= \left(\frac{16}{3} - 2k\right) - \left(2\alpha^2 - \frac{\alpha^3}{3} - k\alpha\right) \quad ..(\text{ii})$

We have R.A: $A = 2(A_1 + A_2)$, So Add (i) and (ii)

$\Rightarrow A = \left(-2\alpha^2 + \frac{\alpha^3}{3} + k\alpha\right) + \left(\frac{16}{3} - 2k\right) - \left(2\alpha^2 - \frac{\alpha^3}{3} - k\alpha\right)$

$= \frac{2\alpha^3}{3} - 4\alpha^2 + 2k(\alpha - 1) + \frac{16}{3}$

$= \frac{2\alpha^3}{3} - 4\alpha^2 + 2(4\alpha - \alpha^2)(\alpha - 1) + \frac{16}{3}$ {From (1)}

$= \frac{-4\alpha^3}{3} + 6\alpha^2 - 8\alpha + \frac{16}{3}$

For Area to be minimum we must have $\frac{dA}{d\alpha} = 0$

So $\frac{dA}{d\alpha} = -4\alpha^2 + 12\alpha - 8$

Now $\frac{dA}{d\alpha} = 0$

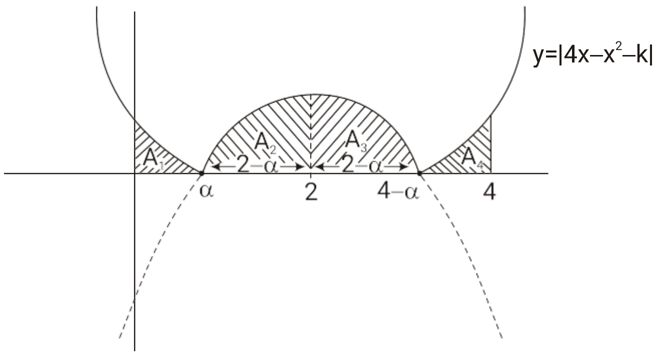
$\Rightarrow -4\alpha^2 + 12\alpha - 8 = 0$

$\Rightarrow \alpha^2 - 3\alpha + 2 = 0$

$\Rightarrow (\alpha - 1)(\alpha - 2) = 0$

$\Rightarrow \alpha = 1, 2$

We can easily verify that $\alpha = 1$ is the point of minima as $\frac{d^2A}{d\alpha^2}\Big|$



$\left.\dfrac{}{d\alpha^2}\right|_{\alpha=1} > 0$

And If $\alpha = 1$

We have $k = \alpha(4 - \alpha) = 1(4 - 1) = 3$

$\therefore$ $k = 3$ is our required answer

64. We can rewrite the given equation

$ax^2 - a(b + c)x + abc + x - d = 0$

$\Rightarrow a(x - b)(x - c) + x - d = 0$

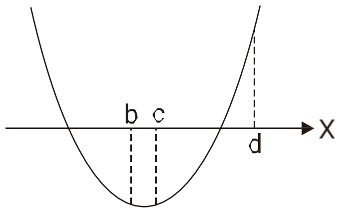
Let $f(x) = a(x - b)(x - c) + x - d$

As $a > 0, y = f(x)$ represents a parabola which opens upwards.

And also, $f(b) = b - d < 0, f(x) = c - d < 0$

And $f(d) = a(d - b)(d - c) > 0$

Thus, $f(x) = 0$ has exactly one root lying in $(-\infty, b)$ and one root in (c, d)



65.
$$I_n = \int_0^{\pi/2} x^n \cos x\, dx$$

$$= x^n \sin x|_0^{\pi/2} - \int_0^{\pi/2} nx^{n-1} \sin x dx$$

$$= \left(\frac{\pi}{2}\right)^n - 0 - (nx^{n-1}(-\cos x))\Big|_0^{\pi/2} + \int_0^{\pi/2} n(n-1)x^{n-2}(-\cos x)dx$$

$$= \left(\frac{\pi}{2}\right)^n - 0 - n(n-1)\int_0^{n/2} x^{n-2} \cos x dx$$

$$I_n = \left(\frac{\pi}{2}\right)^n - n(n-1)I_{(n-2)}$$

$$\sum_{n=2}^{\infty}\left(\frac{I_n}{n!} + \frac{I_{n-2}}{(n-2)!}\right)$$

$$= \sum_{n=2}^{\infty}\left(\frac{\left(\frac{\pi}{2}\right)^n - n(n-1)I_{n-2}}{n!} + \frac{I_{n-2}}{(n-2)!}\right)$$

$$= \sum_{n=2}^{\infty}\left(\left(\frac{\pi}{2}\right)^n \frac{1}{n!}\right)$$

$$= \left(\frac{\pi}{2}\right)^2\frac{1}{2!} + \left(\frac{\pi}{2}\right)^3\frac{1}{3!} + \left(\frac{\pi}{2}\right)^4\frac{1}{4!} + \ldots$$

$$= e^{\pi/2} - 1 - \left(\frac{\pi}{2}\right)$$

66. $\dfrac{dy}{dx} + y = \dfrac{1}{1 + e^{2x}}$

So integrating factor is $e^{\int 1.dx} = e^x$

So solution is $y \cdot ex = \tan^{-1}(e^x) + c$

Now as curve is passing through $(0, \frac{\pi}{2})$ so

$\Rightarrow c = \dfrac{\pi}{4}$

$\Rightarrow \lim (y \cdot e^x) = \lim (\tan^{-1}(e^x) + \frac{\pi}{}) = \frac{3\pi}{}$

68. Equation of the two circles be

$(x - r)^2 + (y - r)^2 = r^2$

i.e., $x^2 + y^2 - 2rx - 2ry + r^2 = 0$ where $r = r_1$ & r_2.

Condition of orthogonality gives

$2r_1r_2 + 2r_1r_2 = r_1^2 + r_2^2$

$\Rightarrow 4r_1r_2 = r_1^2 + r_2^2$ ....(i)

Circle passes through (a, b)

$\Rightarrow a^2 + b^2 - 2ra - 2rb + r^2 = 0$ i.e.

$r^2 - 2r(a + b) + a^2 + b^2 = 0$

$r_1 + r_2 = 2(a + b)$ and $r_1r_2 = a^2 + b^2$

pur in (i) to get $a^2 - 4ab + b^2 = 0$

69. To find the locus, put $z = x + iy$ in the given equation

$$Re\left(\frac{iz + 1}{iz - 1}\right) = 2$$

$$\Rightarrow Re\left(\frac{i(x + iy) + 1}{i(x + iy) - 1}\right) = 2$$

$$\Rightarrow Re\left(\frac{ix + i^2y + 1}{ix + i^2y - 1}\right) = 2$$

$$\Rightarrow Re\left(\frac{ix - y + 1}{ix - y - 1}\right) = 2$$

$$\Rightarrow Re\left(\frac{y - 1 - ix}{y + 1 - ix}\right) = 2$$

$$\Rightarrow Re\left(\left(\frac{(y - 1) - ix}{(y + 1) - ix}\right).\left(\frac{(y + 1) + ix}{(y + 1) + ix}\right)\right) = 2$$

$$\Rightarrow Re\left(\left(\frac{((y - 1) - ix)((y + 1) + ix)}{(y + 1)^2 - i^2x^2}\right)\right) = 2$$

$$\Rightarrow Re\left(\frac{(y^2 - 1) + x^2) + i(x(y - 1 - y - 1)}{(y + 1)^2 + x^2}\right) = 2$$

.............(i)

Real part $\left(\dfrac{(y^2 - 1) + x^2) + i(x(y - 1 - y - 1)}{(y + 1)^2 + x^2}\right) = \left(\dfrac{(y^2 - 1) + x^2)}{(y + 1)^2 + x^2}\right)$

From equation (i)

$$\Rightarrow \left(\frac{(y^2 - 1) + x^2)}{(y + 1)^2 + x^2}\right) = 2$$

$\Rightarrow (y^2 - 1) + x^2 = 2(y + 1)^2 + 2x^2$

$\Rightarrow x^2 + 2y^2 + 4y + 2 - y^2 + 1 = 0$

$\Rightarrow x^2 + y^2 + 4y + 3 = 0$

This is the equation of a circle.

70. $2^{2y} - 2^y + 2^x(1 - 2^x) = 0$

Putting $2^y = t$, we get

$t^2 - t + 2^x(1 - 2^x) = 0$, where $t_1 = 2^{y_1}$ and $t_2 = 2^{y_1}$

$t_1 t_2 = 2^x(1 - 2^x)$

$x\to\infty$ $x\to\infty$ 4 4

67. $\vec{r} = x\vec{a} + y\vec{b} + z(\vec{a}\times\vec{b})$

dot product with $a, b, a\times b$ we get

$4x + y = 2$

$x + y = 8$

$3z = 4\sqrt{3}$

$x = -2, y = 10, z = \frac{4}{\sqrt{3}}$

1 2

$2^{y_1+y_2} = 2^x(1-2^x)$

$y_1 + y_2 = x + \log_2(1-2^x)$

71. Given $\alpha = \sin^{-1}\left(\frac{2x}{1+x^2}\right), x \in [-1,1]$

$\Rightarrow \alpha = 2\tan^{-1}x \ \forall\ x \in [-1,1]$

Now in $[-1,1]$,

$\tan^{-1}x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$\Rightarrow 2\tan^{-1}x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

Hence, range of α is $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Also, $\beta = \cos^{-1}\left(\frac{3\cos\gamma - 4\sin\gamma}{10}\right)$; $\gamma \in [0, 2\pi]$

We know that

$a\sin x + b\cos x \in [-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}]$ If $x \in [0, 2\pi]$

Hence $3\cos\gamma - 4\sin\gamma \in [-5, 5]$ if $\gamma \in [0, 2\pi]$

$\Rightarrow \frac{3\cos\gamma - 4\sin\gamma}{10} \in [-\frac{5}{10}, \frac{5}{10}] = [-\frac{1}{2}, \frac{1}{2}]$

As $\cos^{-1}x$ is a continuous and a decreasing curve

Hence range of $\beta \in [\cos^{-1}(\frac{1}{2}), \cos^{-1}(-\frac{1}{2})]$

$\Rightarrow$ Range of β is $[\frac{\pi}{3}, \frac{2\pi}{3}]$

Also given $\gamma = 2\tan^{-1}(z^2 - 4x + 5)$, $z \in R$

$\Rightarrow \gamma = 2\tan^{-1}((z-2)^2 + 1)$

Now $(z-2)^2 + 1 \geq 1\ \forall z \in R$

i.e. $(z-2)^2 + 1 \in [1, \infty)$

Also, $\tan^{-1}x$ is an increasing function for all $x \in R$

Hence $\tan^{-1}((z-2)^2+1) \in [\tan^{-1}(1), \tan^{-1}(\infty \uparrow)$

$\Rightarrow \tan^{-1}((z-2)^2+1) \in [\frac{\pi}{4}, \frac{\pi}{2})$

$\Rightarrow 2\tan^{-1}((z-2)^2+1) \in [\frac{\pi}{2}, \pi)$

$\therefore$ Range of γ is $[\frac{\pi}{2}, \pi)$

So, we have

$\frac{\pi}{3} \leq \beta \leq \frac{2\pi}{3}$, and

$\frac{\pi}{2} \leq \gamma < \pi$

$\Rightarrow \frac{\pi}{3} + \frac{\pi}{2} \leq \beta + \gamma < \frac{2\pi}{3} + \pi$

$\Rightarrow \frac{5\pi}{6} \leq \beta + \gamma < \frac{5\pi}{3}$

Thus, $(\beta + \gamma)_{\min} = \frac{5\pi}{6}$,

Which is possible if $\beta = \frac{\pi}{3}$ and $\gamma = \frac{\pi}{2}$

Also, α, β, γ are angles of a triangle.

So, $\alpha = \frac{\pi}{6}$, $\beta = \frac{\pi}{3}$ and $\gamma = \frac{\pi}{2}$

Now $\alpha = 2\tan^{-1}x = \frac{\pi}{6}$

$\Rightarrow \tan^{-1}x = \frac{\pi}{12}$

$\Rightarrow x = \tan\frac{\pi}{12} = 2 - \sqrt{3}$

Also, $\beta = \cos^{-1}\left(\frac{3\cos\gamma - 4\sin\gamma}{10}\right) = \frac{\pi}{3}$

$\Rightarrow \gamma + \vartheta = 2\pi$

$\Rightarrow \gamma = 2\pi - \vartheta = 2\pi - \tan^{-1}\frac{4}{3}$

And $\gamma = 2\tan^{-1}(z^2 - 4x + 5) = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}(z^2 - 4x + 5) = \frac{\pi}{4}$

$\Rightarrow z^2 - 4x + 5 = 1$

$\Rightarrow z^2 - 4x + 4 = 0$

$\Rightarrow (z-2)^2 = 0$

$\Rightarrow z = 2$

Now,

$x + \tan y + z = (2 - \sqrt{3}) + \tan\left(2\pi - \tan^{-1}\frac{4}{3}\right) + 2$

Use the rule $\tan(2\pi - \vartheta) = -\tan\vartheta$

$= (2 - \sqrt{3}) - \tan\left(\tan^{-1}\frac{4}{3}\right) + 2$

$= (2 - \sqrt{3}) - \left(\frac{4}{3}\right) + 2$

$= 4 - \frac{4}{3} - \sqrt{3}$

$= \frac{8 - 3\sqrt{3}}{3}$

$= \frac{8 - \sqrt{27}}{3} \equiv \frac{a - \sqrt{b}}{c}$

$\therefore\ a = 8,\ b = 27,\ c = 3$

$\Rightarrow a + b + c = 38$

72. Digits are 1, 2, 3, 4, 5, 7, 9

Multiple of 11 → Difference of sum at even & odd place is divisible by 11.

Let number of the form abcdefg

$\therefore (a + c + e + g) - (b + d + f) = 11x$

$a + b + c + d + e + f = 31$

$\therefore$ either $a + c + e + g = 21$ or 10

$\therefore b + d + f = 10$ or 21

Case −1 :

$a + c + e + g = 21$

$b + d + f = 10$

$(b, d, f) \in \{(1, 2, 7)(2, 3, 5)(1, 4, 5)\}$

$(a, c, e, g) \in \{(1, 4, 7, 9), (3, 4, 5, 9), (2, 3, 7, 9)\}$

$\therefore$ Total number in case-1 $= (3! \times 3)(4!) = 432$

Case −2 :

$a + c + e + g = 10$

$b + d + f = 21$ $(a, b, e, g) \in \{(1,$

$\Rightarrow \cos\frac{\pi}{3} = \frac{3\cos y - 4\sin y}{10}$

$\Rightarrow \frac{1}{2} = \frac{3\cos y - 4\sin y}{10}$

$\Rightarrow \frac{3\cos y}{5} - \frac{4\sin y}{5} = 1$

Let $\cos\vartheta = \frac{3}{5}$ and $\sin\vartheta = \frac{4}{5}$

$\Rightarrow \cos(y + \vartheta) = 1$, where $\tan\vartheta = \frac{4}{3}$

$\Rightarrow \cos(y + \vartheta) = \cos 2\pi$

$2, 3, 4)\}$
(b, d, f) & $\{(5, 7, 9)\}$

$\therefore$ Total number in case-2= $3! \times 4! = 144$

$\therefore$ Total number$144 + 432 = 576$

73. General term,
$T_{r+1} = {}^nC_r(x^2)^{n-r}(-1)^r x^{-r} = {}^nC_r x^{2n-3r}(-1)^r$
Constant term = ${}^nC_r(-1)^r$ if $2n = 3r$ i.e., coefficient of $x = 0$

hence, ${}^{n}C_{2n/3}(-1)^{2n/3} = 15 = {}^{6}C_4$
n=6

74. We have, CF is perpendicular From the center of the ellipse $\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1$ to tangent at the point P.

Let $P(a\cos\vartheta, b\sin\vartheta) \equiv P(4\cos\vartheta, 3\sin\vartheta)$ be a point on the ellipse
Then, the equation of tangent at $P(4\cos\vartheta, 3\sin\vartheta)$ can be given as

$$\frac{x(4\cos\vartheta)}{4^2} + \frac{y(3\sin\vartheta)}{3^2} = 1$$

or $\frac{x\cos\vartheta}{4} + \frac{y\sin\vartheta}{3} = 1$

or $3x\cos\vartheta + 4y\sin\vartheta = 12 \quad ..(i)$

As per the question CF is perpendicular from the center of the ellipse to the line given in eq.(i)

$$\Rightarrow CF = \frac{3(0)\cos\vartheta + 4(0)\sin\vartheta - 12}{\sqrt{(4\sin\vartheta)^2 + (3\cos\vartheta)^2}}$$

$$\Rightarrow CF = \frac{12}{\sqrt{16\sin^2\vartheta + 9\cos^2\vartheta}} \quad ..(ii)$$

Also, equation of normal at point $P(4\cos\vartheta, 3\sin\vartheta)$ can be given as

$$\frac{ax}{\cos\vartheta} - \frac{by}{\sin\vartheta} = a^2 - b^2$$

or $\frac{4x}{\cos\vartheta} - \frac{3y}{\sin\vartheta} = 4^2 - 3^2$

or $\frac{4x}{\cos\vartheta} - \frac{3y}{\sin\vartheta} = 7 \quad ..(iii)$

Now this normal at P meets the major axis (i.e. $y = 0$) at point G
So, put $y = 0$ in eq.(iii) to find point G

$$\Rightarrow \frac{4x}{\cos\vartheta} - \frac{3(0)}{\sin\vartheta} = 7$$

$$\Rightarrow x = \frac{7\cos\vartheta}{4}$$

So, $G\left(\frac{7\cos\vartheta}{4}, 0\right)$

So, we have $P(4\cos\vartheta, 3\sin\vartheta)$ and $G\left(\frac{7\cos\vartheta}{4}, 0\right)$
Using distance formula, we get

$$PG = \sqrt{\left(4\cos\vartheta - \frac{7\cos\vartheta}{4}\right)^2 + (3\sin\vartheta)^2}$$

$$= \sqrt{\left(\frac{9\cos\vartheta}{4}\right)^2 + (3\sin\vartheta)^2}$$

$$= \frac{3}{4}\sqrt{(9\cos^2\vartheta + 16\sin^2\vartheta)} \quad ..(iv)$$

Multiply (ii) and (iv) to get
$CF \times PG = 12 \times \frac{3}{4} = 9 = b^2$

Note: Considering the same question for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, product $CF \times PG = b^2$ and should be remembered as a standard result

75. $e^{f(x)} = \frac{10+x}{10-x}, x \in (-10, 10) \Rightarrow f(x) = \log\left(\frac{10+x}{10-x}\right)$

$$\Rightarrow f\left(\frac{200x}{100+x^2}\right) = \log\left[\frac{10 + \frac{200x}{100+x^2}}{10 - \frac{20x}{100+x^2}}\right] = \log\left[\frac{10(10+x)^2}{10(10-x)^2}\right] = 2\log\left(\frac{10+x}{10-x}\right)$$

$$\Rightarrow f\left(\frac{200x}{100+x^2}\right) = 2f(x)$$

$\Rightarrow k = 2$