1. A hollow cylinder of mass M and radius *R* is rotating about its axis of symmetry and a solid sphere of same mass and radius is rotating about an axis passing through its centre. If torques of equal magnitude are applied to them, then the ratio of angular accelerations produced is

(a) $\frac{2}{5}$ (b) $\frac{5}{2}$ (c) $\frac{5}{4}$ (d) $\frac{4}{5}$

2. When a solid sphere rolls without slipping down an inclined plane making an angle θ with the horizontal, the acceleration of its centre of mass is *a*. If the same sphere slides without friction, its acceleration *a'* will be

(a) $\frac{7}{2}a$ (b) $\frac{5}{7}a$ (c) $\frac{7}{5}a$ (d) $\frac{5}{2}a$

3. When a disc rotates with uniform angular velocity, which of the following is not true?
(a) The sense of rotation remains same.
(b) The orientation of the axis of rotation remains same.
(c) The speed of rotation is non – zero and remains same.
(d) The angular acceleration is non – zero remains same.

4. A circular platform is mounted on a vertical frictionless axle. Its radius is r = 2 m and its moment of inertia *I* = 200 kg m^2. It is initially at rest. A 70 kg man stands on the edge of the platform and begins to walk along the edge at speed v_0 = 1 m s^{-1} relative to the ground. The angular velocity of the platform is
(a) 1.2 rad s^{-1} (b) 0.4 rad s^{-1} (c) 0.7 rad s^{-1} (d) 2 rad s^{-1}

5. A child is standing with his two arms outstretched at the centre of a turntable that is rotating about its central axis with an angular speed . Now, the child folds his hands back so that moment of inertia becomes 3 times the initial value. The new angular speed is

(a) 3 ω_0 (b) $\frac{\omega_0}{3}$ (c) $6\omega_0$ (d) $\frac{\omega_0}{6}$

6. A man stands on a rotating platform with his arms stretched holding a 5 kg weight in each hand. The angular speed of the platform is 1.2 rev s^{-1}. The moment of inertia of the man together with the platform may be taken to be constant and equal to 6 kg m^2. If the man brings his arms close to his chest with the distance of each weight from the axis changing from 100 cm to 20 cm. The new angular speed of the platform is
(a) 2 rev s^{-1} (b) 3 rev s^{-1} (c) 5 rev s^{-1} (d) 6 rev s^{-1}

7. The force $7\hat{i}+3\hat{j}-5\hat{k}$ acts on a particle whose position vector is $\hat{i}-\hat{j}+\hat{k}$ What is the torque of a given force about the origin?

(a) $12\hat{i}+12\hat{j}+10\hat{k}$ (b) $2\hat{i}+10\hat{j}+12\hat{k}$

(c) $2\hat{i}+10\hat{j}+10\hat{k}$ (d) $10\hat{i}+2\hat{j}+\hat{k}$

8. Which of the following statements is incorrect?
(a) A pair of equal and opposite forces with different lines of action is known as couple.
(b) A couple produces rotation without translation.
(c) When we open the lid of a bottle by turning it, our fingers apply a couple to the lid.
(d) Moment of a couple depends on the point about which we take the moment.

9. If $\vec{F}$ is the force acting on a particle having position vector $\vec{r}$ and $\vec{\tau}$ be the torque of this force about the origin, then

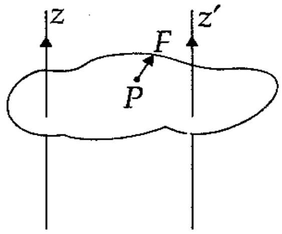
(a) $\vec{r}\,.\,\vec{\tau}>0$ and $F.\vec{\tau}\neq 0$ (b) $\vec{r}\,.\,\vec{\tau}=0$ and $F.\vec{\tau}=0$

(c) $\vec{r}\,.\,\vec{\tau}=0$ and $F.\vec{\tau}\neq 0$ (d) $\vec{r}\,.\,\vec{\tau}\neq 0$ and $F.\vec{\tau}=0$

10. A grindstone has a moment of inertia of 6 kg m^2. A constant torque is applied and the grindstone is found to have a speed of 150 rpm, 10 seconds after starting from rest. The torque is

(a) $3\pi\,N\,m$ (b) $3\,N\,m$ (c) $\frac{\pi}{3}N\,m$ (d) $4\pi\,N\,m$

11. Figure shows a lamina in *x-y* plane. Two axes z and *z'* pass perpendicular to its plane. A force F acts in the plane of lamina at point *P* as shown. Which of the following statements is incorrect?
(The point *P* is closer to z'-axis than the z- axis).



(a) Torque τ caused by F about z axis is along $\hat{k}$

(b) Torque τ' caused by F about z' axis is along $-\hat{k}$

(c) Torque caused by F about z axis is greater in magnitude than that about *z'* axis.

(d) Total torque is given by $\tau = \tau + \tau'$

12. Analogue of mass in rotational motion is

(a) Moment of inertia (b) Torque

(c) Radius of gyration (d) Angular momentum

13. The moment of inertia of a body depends UponThe moment of inertia of a solid sphere of mass *M* and radius *R* about a tangent to the sphere is

(a) $\frac{2}{5}MR^2$ (b) $\frac{6}{5}MR^2$ (c) $\frac{4}{5}MR^2$ (d) $\frac{7}{5}MR^2$

14. The radius of gyration of an uniform rod of length *l* about an axis passing through one of its ends and perpendicular to its length is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2}$

15. Which of the following has the highest moment of inertia when each of them has the same mass and the same radius?

(a) A ring about any of its diameter.

(b) A disc about any of its diameter

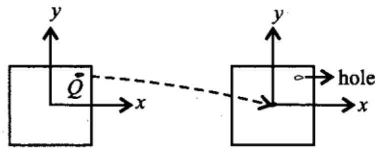
(c) A hollow sphere about any of its diameter.

(d) A solid sphere about any of its diameter.

16. A ballet dancer, dancing on a smooth floor is spinning about a vertical axis with her arms folded with an angular velocity of 20 rad/s. When she stretches her arms fully, the spinning speed decrease in 10 rad/s. If *I* is the initial moment of inertia of the dancer, the new moment of inertia is

(a) 2I (b) 3 I (c) 1/2 (d) I / 3

17. A uniform square plate has a small piece Q of an irregular shape removed and glued to the centre of the plate leaving a hole behind. The moment of inertia about the z – axis is then



(a) Increased (b) Decreased

(c) The same (d) Changed in unpredicted manner

18. The position of a particle is given by $\vec{r} = \hat{i} + 2\hat{j} - \hat{k}$ and its linear momentum is given by . $\vec{p} = 3\hat{i} + 4\hat{j} - 2\hat{k}$ Then its angular momentum about the origin is perpendicular to

(a) x-axis (b) y-axis (c) z-axis (d) yzplane

19. A solid cylinder of mass 20 kg and radius 20 cm rotates about its axis with a angular speed 100 rad s^{-1}. The angular momentum of the cylinder about its axis is

(a) 40 Js (b) 400 Js (c) 20 J s (d) 200 J s

20. Which of the following principles a circus acrobat employs in his performance?

(a) Conservation of energy

(b) Conservation of linear momentum

(c) Conservation of mass
(d) Conservation of angular momentum

21. Two bodies have their moments of inertia *I* and *21* respectively about their axis of rotation. If their kinetic energies of rotation are equal, their angular momenta will be in the ratio
(a) 1:2 (b) $\sqrt{2}:1$ (c) $1:\sqrt{2}$ (d) 2 : 1

22. In the question number 58, if wheel starts from rest, what is the kinetic energy of the wheel when 2 m of the cord is unwound?
(a) 20 J (b) 25 J (c) 45 J (d) 50 J

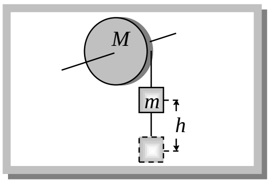
23. A child is standing with folded hands at the centre of a platform rotating about its central axis. The kinetic energy of the system is K. Now, the child stretches his arms so that moment of inertia of the system doubled. Now, the kinetic energy of the system is
(a) $\frac{k}{4}$ (b) $\frac{k}{2}$ (c) $2K$ (d) 4K

24. The moments of inertia of two rotating bodies *A* and *B* are I_A and I_B $(I_A > I_B)$. If their angular momenta are equal, then
(a) Kinetic energy of *A* = Kinetic energy of *B*
(b) Kinetic energy of A > Kinetic energy of *B*
(c) Kinetic energy of A < Kinetic energy of *B*
(d) Kinetic energy of the two bodies cannot be compared with the given data

25. A wheel of mass 5 kg and radius 0.40 m isrolling on a road without sliding with angular velocity 10 rad s^{-1}. The moment of inertia of the wheel about the axis of rotation is 0.65 kg m^2. The percentage of kinetic energy of rotation in the total kinetic energy of the wheel is
(a) 22.4% (b) 11.2% (c) 88.8 % (d) 44.8 %

26. A solid cylinder rolls up an inclined plane of inclination θ with an initial velocity v. How far does the cylinder go up the plane?
(a) $\frac{3v^2}{2g\sin\theta}$ (b) $\frac{v^2}{4g\sin\theta}$ (c) $\frac{3v^2}{g\sin\theta}$ (d) $\frac{3v^2}{4g\sin\theta}$

27. A massless string is wrapped round a disc of mass *M* and radius *R*. Another end is tied to a mass *m* which is initially at height *h* from ground level as shown in the fig. If the mass is released then its velocity while touching the ground level will be
(a) $\sqrt{2gh}$
(b) $\sqrt{2gh}\frac{M}{m}$
(c) $\sqrt{2ghm/M}$
(d) $\sqrt{4mgh/2m+M}$



28. In the above problem the angular velocity of the cylinder, after the masses fall down through distance *h*, will be
(a) $\frac{1}{R}\sqrt{8mgh/(M+4m)}$ (b) $\frac{1}{R}\sqrt{8mgh/(M+m)}$
(c) $\frac{1}{R}\sqrt{mgh/(M+m)}$ (d) $\frac{1}{R}\sqrt{8mgh/(M+2m)}$

29. A solid cube of side *l* is made to oscillate about a horizontal axis passing through one of its edges. Its time period will be
(a) $2\pi\sqrt{\frac{2\sqrt{2}}{3}\frac{l}{g}}$ (b) $2\pi\sqrt{\frac{2}{3}\frac{l}{g}}$
(c) $2\pi\sqrt{\frac{\sqrt{3}}{2}\frac{l}{g}}$ (d) $2\pi\sqrt{\frac{2}{\sqrt{3}}\frac{l}{g}}$

30. A ring whose diameter is 1 *meter*, oscillates simple harmonically in a vertical plane about a nail fixed at its circumference. The time period will be

(a) $1/4$ *sec* (b) $1/2$ *sec*

(c) 1 *sec* (d) 2 *sec*

1. (a): Moment of inertia of hollow cylinder about its axis of symmetry, $I_1 = MR^2$

Moment of inertia of solid sphere about an axis passing its centre, $I_2 = \frac{2}{5}MR^2$

Let α_1 and α_2 beangular accelerations produced in the cylinder and the sphere repectively on applying same torque τ in each case. Then

$\alpha_1 = \frac{\tau}{I_1}$ (As) $\tau = I\alpha$) and $\alpha_2 = \frac{\tau}{I_2}$

Their corresponding ration is

$$\frac{\alpha_1}{\alpha_2} = \frac{I_2}{I_1} = \frac{\frac{2}{5}MR^2}{MR^2} = \frac{2}{5}$$

2. (c): Acceleration of the solid sphere when it rolls without slipping down an inclined plane is.

$$a = \frac{g\sin\omega}{1+\frac{I}{MR^2}}$$

For a solid sphere, $I = \frac{2}{5}MR^2$

$$\therefore a = \frac{g\sin\theta}{1+\frac{2}{5}} = \frac{5}{7}g\sin\theta \qquad \ldots(i)$$

Acceleration ofj the same sphere when it slides when it slides without friction down an same inclined plane is , $a' = g\sin\theta$ …(ii)

Divide (ii) by (i), we get,

$$\frac{a'}{a} = \frac{7}{5} \text{ or } a' = \frac{7}{5}a$$

3. (d) When a disc rotates with uniform angular velocity, angular accelerations of the disc is zero, Hence, options (d) is not true.

4. (c): As the system is initially at rest, therefore initial angular momentum, $L_i = 0$.

According to the law of conservation of angular momentum final angular momentum, $L_f = 0$

$\therefore$ angular momentum ofj man = angular momentum of platform in opposite direction

i.e., $mv_0r = I\omega$

or, $\omega = \frac{mv_0r}{I} = \frac{70(1.0)(2)}{200} = 0.7\,\text{rad}\,s^{-1}$

5. (b): Here, Initial angular spped, $\omega_i = \omega_0$

Initial moment ofj inertia, = I_i

Final moment of inertia, $I_f = 3I_i$

According to the law of conservation of angular momentum, we get,

$$L_i = L_f$$

$$I_i\omega_i = I_f\omega_f$$

$$\omega_f = \frac{I_i\omega_i}{I_f} = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{I_i}{3I_i}\right)\omega_i \qquad (\because I_f = 3I_i)$$

$$= \frac{\omega_i}{3} = \frac{\omega_0}{3}$$

6. (b):Initial moment of inertia,

$I_1 = 6 + 2\times 5\times (1)^2 = 16 \text{kg m}^2$

Initial angular velocity,

$\omega_1 = 1.2 \text{revs}^{-1}$

Initial angular momentum is

$L_1 = I_1\omega_1$

Final moment of inertia,

$I_2 = 6 + 2\times 5\times (0.2)^2 = 6.4 \text{kg m}^2$

Final angularspeed, = ω_2

Final angular momentum is,

$l_2 = I_2\omega_2$

According to law of conservation of angular momentum,

$L_1 = L_2$ or $I_1\omega_1 = I_2\omega_2$

$$\omega_2 = \frac{I_1\omega_1}{I_2} = \frac{(16\text{kgm}^2)(1.2\text{revs}^{-1})}{(6.4\text{kg m}^2)} = 3\text{revs}^{-1}$$

7. (a) Here, $\vec{r} = \hat{i} - \hat{j} + \hat{k}$

$\vec{F} = 7\hat{i} + 3\hat{j} - 5\hat{k}$

Torque, $\vec{\tau} = \vec{r}\times\vec{F}$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 7 & 3 & -5 \end{vmatrix} = \hat{i}(5-3) + \hat{j}(7-(-5)) + \hat{k}(3-(-7)) \text{ or } \vec{\tau} = 2\hat{i} + 12\hat{j} + 10\hat{k}$$

8. (d) Moment of a couple depends on the point about which we take the moment.

9. (b) Torque is always perpendicular to $\vec{F}$ as well as $\vec{r}$.

$\therefore \vec{r}.\vec{\tau} = 0$ as well as $\vec{F}.\vec{\tau} = 0$

10. (a): Here, I = 6 kg m^2 t = 10 s

$\omega_0 = 0$

$$\upsilon = 150\text{rpm} = \frac{150}{60}\text{rps} = \frac{5}{2}\text{rps}$$

$$\omega = 2\pi\upsilon = 2\pi\times\frac{5}{2} = 5\pi\text{rad s}^{-1}$$

$$\alpha = \frac{\omega-\omega_0}{t} = \frac{5\pi-0}{10} = \frac{\pi}{2}\text{rad s}^{-2}$$

$$\therefore \text{ Torque, } \tau = I\alpha = 6\times\frac{\pi}{2} = 3\pi\text{N m}$$

11. (d) The directions of $\vec{\tau}$ is given by right handed screw rule.

According to this rule, the directions of $\vec{\tau}$ caused by $\vec{F}$ about z axis is along $\hat{k}$.

Hence, options (a) is correct.

Again, according to this rule, the directions of $\vec{\tau}$ caused by $\vec{F}$ about z axis is along $-\hat{k}$.

Hence ,option (b) is correct.

As $\vec{\tau} = \vec{r}\times\vec{F}$ and P is closer z` axis, therefore $\vec{\tau}$ caused by $\vec{F}$ about z axis is greater in magnitude than that about z` axis.

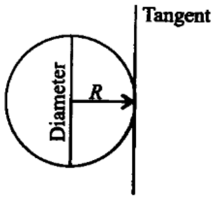
Hence option (c) is also correct.

It is meaningless to add torques about two different axes.

Hence, options (d) is incorrect.

12. (a) Analogue of mass in rotational motions is moment of inertia.

13. (d)



Moment of inertia of the solid sphere of mass M and radius R about any diameter is

$I_{diamete} = \frac{2}{5}MR^2$

According to theorem of parallel axes

$I_{tan\,gent} = I_{diameter} + MR^2, I_{Li'kZk;} = I_{Okl} + MR^2 = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$

14. (c): Moment of inertia ofrodof mass Mand length l about its axis passing through one of its ends andperpendicular to it is.

$I = \frac{1}{3}Ml^2$

As: $I = Mk^2$ where k is the radius of the gyration $\therefore Mk^2 = \frac{1}{3}Ml^2$ or $k = \frac{1}{\sqrt{3}}$

15. (c): Let M and R be the mass and radius respectively. Moment ofinertia of a ring about any of its diameter is. $I_{ring} = \frac{1}{2}MR^2$

Moment of inertia of a disc about any of its diameter is

$I_{disc} = \frac{1}{4}MR^2$

Moment of inertia of a hollow sphere about any of its diameter is.

$I_{hollowsphere} = \frac{2}{3}MR^2$

Moment of inertia of a solid sphere about any of its diameter is.

$I_{solidsphere} = \frac{2}{5}MR^2$

Thus, $I_{hollow\ sphere}$ is largest.

16. (a) Here, angular momentum is conserved.
Initial angular momentum = final angular
Momentum
$I \times 20 = I' \times 10$]
Where I' is new moment of inertia, $I' = 2I$

17. (b) According to the theorem of perpendicular axes $I_z = I_x + I_y$

With the hole, I_x and I_y both decreases. Gluing the removed piece at the centre of square plate does not affect I_z Hence, I_z decreases overall.

18. (a) Here, $\vec{r} = \hat{i} + 2\hat{j} - \hat{k}, \vec{p} = 3\hat{i} + 4\hat{j} - 2\hat{k}$

$$\vec{L} = \vec{r} \times \vec{P} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & 4 & -2 \end{vmatrix}$$

$$= \hat{i}(-4+4) + \hat{j}(-3+2) + \hat{k}(4-6) = 0\hat{i} - 1\hat{j} - 2\hat{k}$$

$\vec{L}$. has components along – y axis and –z axis but it has no component along in the x-axis. The angular momentum $\vec{L}$. is in yz-plane. i.e., perpendicular to x – axis.

19. (a): Here, M = 20 kg

R = 20 cm = 20 × 10^{-2} m, $\omega = 100$ rad s^{-1}

Moment of inertia of the solid cylinder about its axis is

$$I = \frac{MR^2}{2} = \frac{(20\,kg)(20 \times 10^{-2}\,m)^2}{2} = 0.4\,kg\,m^2$$

Angular momentum of the cylinder about its axis is L = $I\omega = (0.4\,kg\ m^2)(100\,rad\ s^{-1}) = 40\ J\,s$

20. (d): A circus acrobat employing the principle of conservation of angular momentum.

21. (c): As, $K_{R_1} = K_{R_2}$

$$\therefore \frac{1}{2} I_1 \omega_1^2 = \frac{1}{2} I_2 \omega_2^2$$

or $\dfrac{\omega_1}{\omega_2} = \sqrt{\dfrac{I_2}{I_1}}$ …(i)

$$\frac{L_1}{L_2} = \frac{I_1 \omega_1}{I_2 \omega_2} = \frac{I_1}{I_2}\sqrt{\frac{I_2}{I_1}} = \sqrt{\frac{I_1}{I_2}} \quad \text{(Using (i))}$$

$$\frac{L_1}{L_2} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

22. (d): Here, R = 20 cm = 0.2 m, M = 20 kg

As τ = FR = (25 N) (0.2m) = 5 N m

Moment of inertia of flywheel about its axis is

$$I = \frac{MR^2}{2} = \frac{(20kg)(0.2m)^2}{2} = 0.4\,kg\,m^2$$

As: $\tau = I\alpha$

$\therefore$ Angular acceleration of the wheel.

$$\alpha = \frac{\tau}{I} = \frac{5\,Nm}{0.4\,kg\ m^2} = 12.5\,rad\,s^{-2}$$

Angular displacement ofwheel.

$$\theta = \frac{Length of\ unwound string}{radius\ of\ the\ wheel} = \frac{2m}{0.2\,m} = 10 rad$$

Let ω bi final angular velocity.

As: $\omega^2 = \omega_0^2 + 2\alpha\theta$

Since the wheel starts from rest, therefore $\omega_0 = 0$

$\therefore \omega^2 = 2 \times (12.5\,rad\ s^{-2})$ (10 rad) = 250 $rad^2\ s^{-2}$

$\therefore$ Kinetic energy gained, $K = \dfrac{1}{2} I\omega^2$

$$K = \frac{1}{2} \times 0.4\,kg\ m^2 \times 250\,rad^2\ s^{-2} = 50\,J$$

23. (b): Initial kinetic energy, $K = \frac{1}{2}I\omega^2 = \frac{K}{2}$ ..(i)

According to the principle of conservation of angular momentum, $I\omega$ = constant As I is doubled, ω becomes half.
So final kinetic energy.

$$K' = \frac{1}{2}(2I)\left(\frac{\omega}{2}\right)^2 = \frac{1}{4}I\omega^2 = \frac{K}{2} \quad \text{(Using (i))}$$

24. (c): $I_A\omega_A = I_B\omega_B$ (Given)

$$\therefore \frac{\omega_A}{\omega_B} = \frac{I_B}{I_A} \quad \ldots(i)$$

Kineticenergy, $= \frac{1}{2}I\omega^2$

$$\therefore \frac{(K.E)_A}{(K.E)_B} = \frac{\frac{1}{2}I_A\omega_A^2}{\frac{1}{2}I_B\omega_B^2}$$

$$= \frac{I_A}{I_B} \times \left(\frac{I_B}{I_A}\right)^2 \quad \text{(Using(i))}$$

$$= \frac{I_B}{I_A}$$

As $I_A > I_B$ (Given)

$\therefore (K.E)_A < (K.E)_s$

25. (d): Here, m = 5 kg, I = 0.65 kg m^2

$\omega = 10 \text{rad s}^{-1}, R = 0.40\text{m}$

Linear velocity,

$v = R\omega = 0.40 \times 10 = 4\text{m s}^{-1}$

Translation KE,

$= \frac{1}{2}mv^2 = \frac{1}{2} \times 16 = 40\text{J}$ Rotational KE,

$= \frac{1}{2}I\omega^2 = \frac{1}{2} \times 0.65 \times 100 = 32.5\text{J}$

Total KE = Translational KE + Rotational KE,

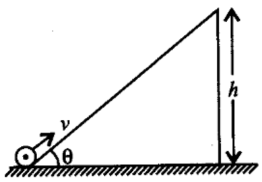
$= 40 + 32.5 = 72.5\text{J}$

Perecentage of rotational KE,

$$= \frac{\text{Rotational KE}}{\text{Total KE}} \times 100$$

$$= \frac{32.5}{72.5} \times 100 = 44.8\%$$

26. (d):



Let the cylinder go up the plane upto a height h let M and R be the mass and radius of the cylinder respectively. According to law of consesrvation of mechanincal energy we get

$$\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = Mgh$$

$$\frac{1}{2}Mv^2 + \frac{1}{2}\frac{MR^2}{2}\omega^2 = Mgh \left(\because \text{For a solid cylinder}, I = \frac{1}{2}MR^2\right) \frac{1}{2}Mv^2 + \frac{1}{4}MR^2\omega^2 = Mgh$$

$$\frac{1}{2}Mv^2 + \frac{1}{4}Mv^2 = Mgh \quad (\because v = R\omega)\ \frac{3}{4}Mv^2 = Mgh$$

$$h = \frac{3v^2}{4g} \qquad \ldots(i)$$

Let s be distanvetravelled by thecylinder up the plane then

$$\sin\theta = \frac{h}{s} \text{ or } s = \frac{h}{\sin\theta} = \frac{3v^2}{4g\sin\theta} \quad \text{(Using(i))}$$

27. D

28. A

29. A

30. (d) $T = 2\pi\sqrt{\frac{2R}{g}} = 2\pi\sqrt{\frac{1}{g}} = 2\ sec$ [As diameter $2R = 1\ meter$ given]